

# Algebra Cheat Sheet

## Basic Properties & Facts

### Arithmetic Operations

$$ab + ac = a(b + c) \quad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\left(\frac{a}{b}\right) \frac{c}{d} = \frac{ac}{bd} \quad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c, \quad a \neq 0 \quad \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) = \frac{ac}{bd}$$

### Exponent Properties

$$a^n a^m = a^{n+m} \quad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$(a^n)^m = a^{nm} \quad a^0 = 1, \quad a \neq 0$$

$$(ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \quad a^{\frac{p}{m}} = \left(a^{\frac{1}{m}}\right)^p = \left(a^n\right)^{\frac{1}{m}}$$

### Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = a, \text{ if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even}$$

### Properties of Inequalities

If  $a < b$  then  $a + c < b + c$  and  $a - c < b - c$

If  $a < b$  and  $c > 0$  then  $ac < bc$  and  $\frac{a}{c} < \frac{b}{c}$

If  $a < b$  and  $c < 0$  then  $ac > bc$  and  $\frac{a}{c} > \frac{b}{c}$

### Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0 \quad |-a| = |a|$$

$$|ab| = |a||b| \quad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a+b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

### Distance Formula

If  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a+bi) + (c+di) = a+c + (b+d)i$$

$$(a+bi) - (c+di) = a-c + (b-d)i$$

$$(a+bi)(c+di) = ac - bd + (ad+bc)i$$

$$(a+bi)(a-bi) = a^2 + b^2$$

$$|a+bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a+bi)} = a-bi \quad \text{Complex Conjugate}$$

$$\overline{(a+bi)}(a+bi) = |a+bi|^2$$

## Logarithms and Log Properties

### Definition

$y = \log_b x$  is equivalent to  $x = b^y$

### Example

$\log_5 125 = 3$  because  $5^3 = 125$

### Special Logarithms

$\ln x = \log_e x$  natural log

$\log x = \log_{10} x$  common log

where  $e = 2.718281828\mathbf{K}$

### Logarithm Properties

$$\log_b b = 1 \quad \log_b 1 = 0$$

$$\log_b b^x = x \quad b^{\log_b x} = x$$

$$\log_b (x^r) = r \log_b x$$

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$$

The domain of  $\log_b x$  is  $x > 0$

## Factoring and Solving

### Factoring Formulas

$$x^2 - a^2 = (x+a)(x-a)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If  $n$  is odd then,

$$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \mathbf{L} + a^{n-1})$$

$$x^n + a^n$$

$$= (x+a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \mathbf{L} + a^{n-1})$$

### Quadratic Formula

Solve  $ax^2 + bx + c = 0$ ,  $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If  $b^2 - 4ac > 0$  - Two real unequal solns.

If  $b^2 - 4ac = 0$  - Repeated real solution.

If  $b^2 - 4ac < 0$  - Two complex solutions.

### Square Root Property

If  $x^2 = p$  then  $x = \pm\sqrt{p}$

### Absolute Value Equations/Inequalities

If  $b$  is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

### Completing the Square

Solve  $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the  $x^2$

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of  $x$ , square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for  $x$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

## Functions and Graphs

### Constant Function

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point  $(0, a)$ .

### Line/Linear Function

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point  $(0, b)$  and slope  $m$ .

#### Slope

Slope of the line containing the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

#### Slope – intercept form

The equation of the line with slope  $m$  and y-intercept  $(0, b)$  is

$$y = mx + b$$

#### Point – Slope form

The equation of the line with slope  $m$  and passing through the point  $(x_1, y_1)$  is

$$y = y_1 + m(x - x_1)$$

### Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at  $(h, k)$ .

### Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at  $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ .

### Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if  $a > 0$  or left if  $a < 0$  and has a vertex at  $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$ .

### Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius  $r$  and center  $(h, k)$ .

### Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center  $(h, k)$  with vertices  $a$  units right/left from the center and vertices  $b$  units up/down from the center.

### Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at  $(h, k)$ , vertices  $a$  units left/right of center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

### Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at  $(h, k)$ , vertices  $b$  units up/down from the center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

## Common Algebraic Errors

| Error   | Reason/Correct/Justification/Example   |
|---|--|
| $\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$                             | Division by zero is undefined!   |
| $-3^2 \neq 9$   | $-3^2 = -9$ , $(-3)^2 = 9$ Watch parenthesis!  |
| $(x^2)^3 \neq x^5$  | $(x^2)^3 = x^2 x^2 x^2 = x^6$  |
| $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$                            | $\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$   |
| $\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$                                  | A more complex version of the previous error.  |
| $\frac{a+bx}{a} \neq 1+bx$  | $\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$<br>Beware of incorrect canceling!   |
| $-a(x-1) \neq -ax-a$  | $-a(x-1) = -ax+a$<br>Make sure you distribute the “-“!   |
| $(x+a)^2 \neq x^2+a^2$  | $(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$   |
| $\sqrt{x^2+a^2} \neq x+a$   | $5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4=7$  |
| $\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$                                     | See previous error.  |
| $(x+a)^n \neq x^n+a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$ | More general versions of previous three errors.  |
| $2(x+1)^2 \neq (2x+2)^2$  | $2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$<br>$(2x+2)^2 = 4x^2+8x+4$<br>Square first then distribute!  |
| $(2x+2)^2 \neq 2(x+1)^2$  | See the previous example. You can not factor out a constant if there is a power on the parenthesis!  |
| $\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$                                    | $\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$<br>Now see the previous error.  |
| $\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$                    | $\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$ |
| $\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$                    | $\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$ |