MTH 253 Common Power Series Compendium

Compendium of the commonly used power series we see this term.

- $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ...$ $\qquad I = (-1, 1)$

- $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + ...$ $\qquad I = (-\infty, \infty)$

- $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + ...$ $\qquad I = (-\infty, \infty)$

- $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + ...$ $\qquad I = (-\infty, \infty)$

- $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + ...$ $\qquad I = [-1, 1]$

- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + ...$ $\qquad I = (-1, 1]$

- $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \dfrac{k(k-1)}{2!}x^2 + \dfrac{k(k-1)(k-2)}{3!}x^3 + ...$ $\qquad R = 1$

Note: You need to have the first 4 of these memorized. I will provide the last 3 for you on the exams.
Also note that the binomial expansion's interval of convergence depends on the value of k, however its radius of convergence is always 1 so it is simply a question of whether or not it converges for at -1 or 1 which can be checked for each value of k.