Math 253 Convergence Tests Summary

- Divergence Test: Given $\sum a_n$, if a_n does not go to zero, then $\sum a_n$ has no chance to converge. If a_n does go to zero, there is a chance.
- Geometric Series Test: Given $\sum\limits_{n=0}^{\infty} = a + ar + ar^2 + ar^3 + ...$, called a geometric series, it converges if and only if $|r| < 1$. If it does converge it then equals $\frac{a}{1-r}$.
- Integral Test: Given $\sum\limits_{n=a}^{\infty} f(n)$, with $f(n)$ positive, decreasing, and going to zero, then $\sum\limits_{n=a}^{\infty}$ converges if and only if $\int\limits_{a}^{\infty} f(x)dx$ converges.
- P-test: A series of the form $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.
- Alternating series test: Given $\sum(-1)^n a_n$, If a_n is eventually positive, decreasing, and going to zero, then $\sum(-1)^n a_n$ converges. If a_n does not go to zero, the series is divergent by the divergence test. If the series does converge, then the k^{th} partial sum, $\sum\limits^{k}(-1)^n a_n$ has error which is less than a_{k+1}.
- Ratio Test: Given $\sum a_n$, if $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, the series converges. If $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ the series diverges. If $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the ratio test is inconclusive.
- Comparison Tests:
  - Direct Comparison: If you can bound a series above and below by either a number or a convergent series (or one of each), then the series you bounded must converge. If you can bound a series above a divergent series, the series in question must diverge.
  - Limit Comparison: If you know a series should behave like another series, but can't bound it correctly, as above, use the limit comparison test which says that: Given $\sum a_n$ and $\sum b_n$, if $\lim\limits_{n\to\infty} \frac{a_n}{b_n}$ is a positive real number, the both series converge or both series diverge (they behave the same).