Math 252 PSU Exam 1 Review　　　　Name: ____________________

NOTE: You will be allowed your calculator for the exam. However, you will also be expected to show your work/reasoning in order to earn ANY credit at all. Meaning, answers alone will earn zero points.

1. Compute the following sums by first writing them in expanded form.

   a. $\sum_{i=1}^{5} 6$

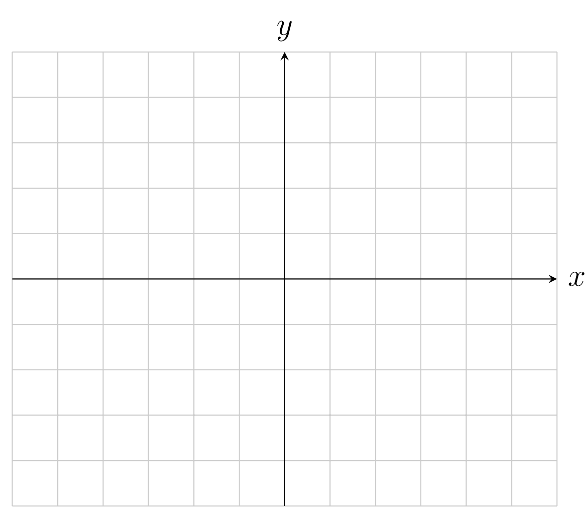
   b. $\sum_{j=3}^{8} (5j+2)$

2. Write the following sums using Sigma Notation (Summation Notation).

   a. $2+4+8+16+32$

   b. $(5+4)+(8+9)+(11+16)+(14+25)+(17+36)$

3. Appoximate the area under the curve $f(x) = x - x^2$ over the interval $[-1, 1]$ using R_6. Write your computation in expanded and Sigma notation.



4. Given these formulas:

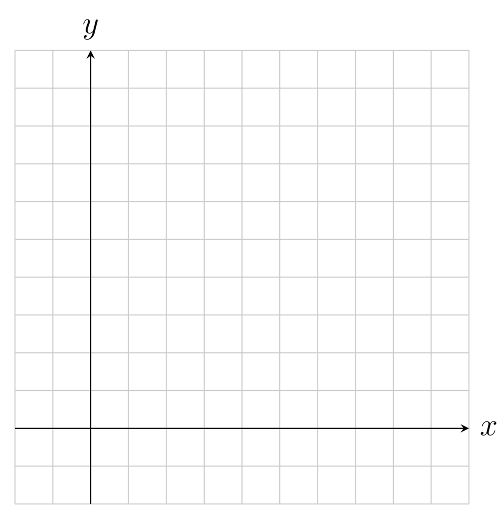
- $\sum_{i=1}^{N} 1 = N$
- $\sum_{i=1}^{N} i = \dfrac{N(N+1)}{2}$
- $\sum_{i=1}^{N} i^2 = \dfrac{N(N+1)(2N+1)}{6}$
- $\sum_{i=1}^{N} i^3 = \left(\dfrac{N(N+1)}{2}\right)^2$

Simplify the expression

$$\sum_{i=0}^{N-1} \left[\frac{-32}{N^4} i^3 + \frac{16}{N^2} i + \frac{2}{N}\right]$$

so as to remove the sigma. Be careful to note the index starting and ending values.

5. Determine a formula for the approximation of the area under the curve $f(x) = x^2 - 4x + 4$ over the interval $[1, 4]$ using L_N. Then find the exact area by finding $\lim_{N \to \infty} L_N$.



Facts:

- $\int\limits_0^v b \; c \; dx = cb$
- $\int\limits_0^b \; x \; dx = \frac{1}{2}b^2$
- $\int\limits_0^b \; x^2 \; dx = \frac{1}{3}b^3$

6. Evaluate the following integrals using the above rules and the fact that integration is a linear operator.

a. $\int\limits_0^2 \; 12y^2 + 6y - 3 \; dy$

c. $\int\limits_2^{-5} \; 9 - 3x^2 \; dx$

b. $\int\limits_{-4}^3 \; 3x^2 + 1 \; dx$

d. $\int\limits_3^4 \; f(x) \; dx$ where $\int\limits_0^1 \; f(x) \; dx = 2$
$\int\limits_0^3 \; f(x) \; dx = 5$ and $\int\limits_1^4 \; f(x) \; dx = 7$

7. Evaluate the following indefinite integrals.

a. $\int 2x + \sec^2(3x)\ dx$

c. $\int \dfrac{12 - 3z}{\sqrt{z^3}}\ dz$

b. $\int z^{-3/5} - z^{1/3} + z^{1/4}\ dz$

d. $\int 8\cos(x) - 3e^{2x-1}\ dx$

8. Solve the initial value problem $f''(x) = x^2 - 2,\ f'(1) = 0,\ f(1) = 2$

9. Suppose a ball is dropped and it falls for 3 seconds before hitting the ground. Determine how far it falls, assuming an acceleration of gravity of $-9.8\ \text{m/s}^2$ and no wind resistance.

10. Evaluate the definite integrals using FTC 1.

a. $\int_{-1}^{3} 2u^5 + 3u^2 - 4u + 1 \; du$

c. $\int_{\pi/4}^{5\pi/4} \cos(3x) \; dx$

b. $\int_{1}^{2} \frac{10t^{4/3} - 8t^{1/3}}{t^2} \; dt$

d. $\int_{2}^{6} x^2 + \frac{5}{x} \; dx$

11. Calculate the following derivatives:

a. $\frac{d}{ds} \int_{-2}^{s} \cos^{-1}(3u + u^2) \; du$

b. $\frac{d}{dx} \int_{1}^{2x^3} \frac{3}{t+5} dt$

12. A population of rabbits is increasing at a rate of $100 + 2t + .3t^2$ rabbits per month (t in months). Find the increase in the number of rabbits between the 3rd and 5th months.

13. The rate (in liters per second) at which water enters a tank is recorded ten second intervals. Compute the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first minute.

t (sec)	0	10	20	30	40	50	60
r (liters/sec)	.5	.4	.45	.4	.6	.8	.7

14. Use substitution to evaluate the following integrals.

a. $\int \frac{5}{x(\ln(x))^3} dx$

d. $\int_0^{\pi/9} \sec^2(3x) dx$

b. $\int \frac{3x^2 + x}{(4x^3 + 2x^2)^3} dx$

e. $\int_{-\pi/12}^{\pi/9} \tan^2(3x) \sec^2(3x) dx$

c. $\int_1^3 \frac{x+4}{x^2 + 8x + 3} dx$

f. $\int_0^{\pi/3} \cos^3(t) dt$

15. Evaluate the following integrals using whatever method necessary.

a. $\int_1^4 \frac{dx}{4x^2+1}$

b. $\int_1^2 \frac{\sin(\ln(t))}{t}dt$

c. $\int_{-1/3}^{1/2} \frac{dt}{\sqrt{9-16t^2}}$

d. $\int 3^{5x}dx$

e. $\int \frac{3^{5x}}{\sqrt{3^{5x}-1}}dx$

f. $\int \frac{(x^3-x^{-1})^2}{x^3}dx$