Name: _____

1. Show that every member of the family of functions

$$y = \frac{1 + ce^t}{1 - ce^t}$$

is a solution of the differential equation $y' = \frac{1}{2}(y^2 - 1)$. And then find a solution of the differential equation $y' = \frac{1}{2}(y^2 - 1)$ that satisfies the initial condition $y(0) = 2$
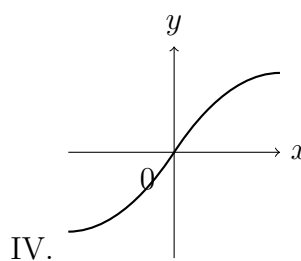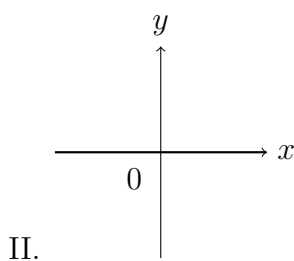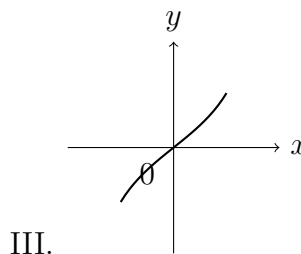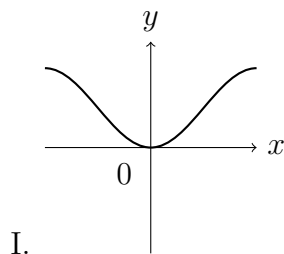
2. Match the differential equations with the solution graphs labeled I-IV. Give reasons for your answer that ensure that your choice is the only possible correct answer. **By elimination is NOT a reason!**.

   a. $y' = 1 + x^2 + y^2$

   c. $y' = \dfrac{1}{1 + e^{x^2 + y^2}}$
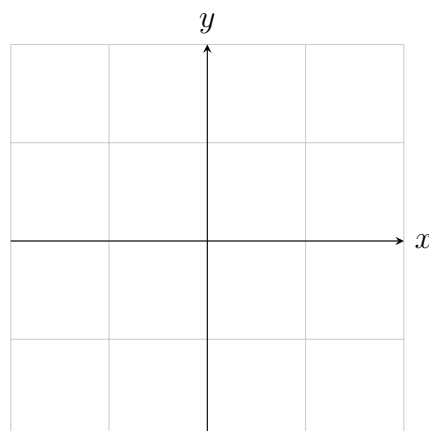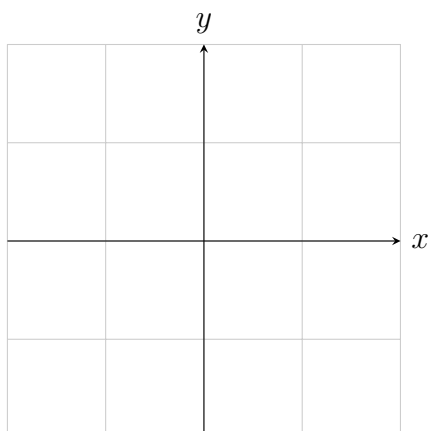
   b. $y' = xe^{-x^2 - y^2}$

   d. $y' = \sin(xy)\cos(xy)$

I.

III.

II.

IV.

3. Sketch the direction field of the differential equation then use it to sketch a solution curve that passes through the given point.

   a. $y' = xy - x^2$, $(0, 1)$
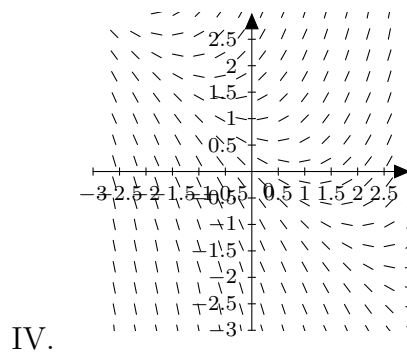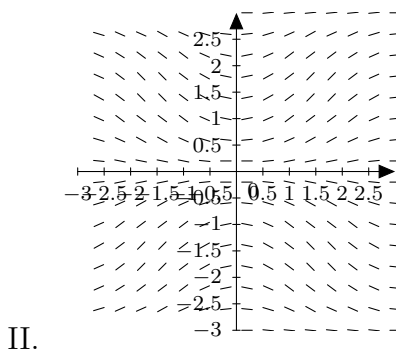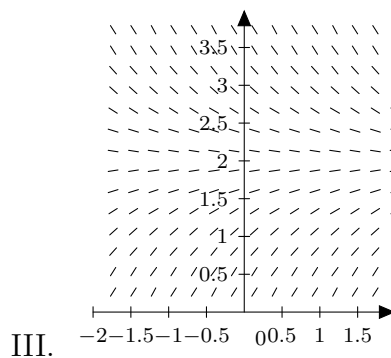
   b. $y' = x + y^2$, $(0, 0)$

4. Match the differential equation with its direction field (labeled I-IV). Give reasons for your answer that ensure that your choice is the only possible correct answer. **By elimination is NOT a reason!**.

(a) $y' = 2 - y$

(c) $y' = x + y - 1$

(b) $y' = x(2 - y)$

(d) $y' = \sin(x)\sin(y)$

I.

III.

II.

IV.
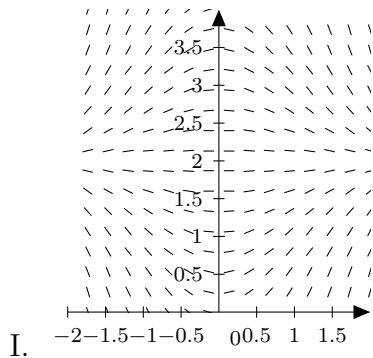
calc   5. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = xy - x^2$, $y(0) = 1$.

6. Use separation to determine solutions to the following initial value differential equations.

a. $y' = \dfrac{xy \sin(x)}{y+1}$, $y(0) = 1$

b. $\dfrac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$

calc   7.   A bottle of soda pop at room temperature (72°F) is placed in a refrigerator where the temperature is 44°F. After half an hour the soda pop has cooled to 61°F. Set up a differential equation modeling this situation and then solve it to find an explicit function $T(t)$ for the temperature (in Fahrenheit) t minutes after the soda has been placed in the refrigerator. What will the soda's temperature be after an hour? When will the soda hit a temperature of 50°F?

8. Solve the following linear first-order differential equations initial value problems.

a. $y' + 3y = e^{2x}$, $y(0) = -1$

b. $y' + y = \sin(x)$, $y(0) = 1$

calc 9. A tank contains 1000L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank (a) after t minutes and (b) after one hour?