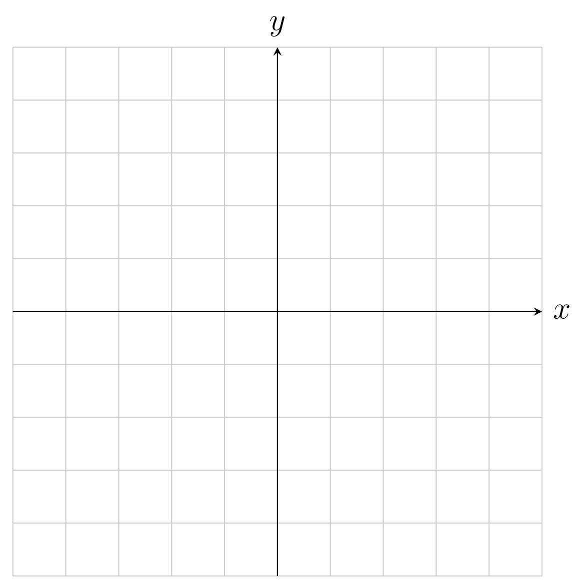
MTH 256 Exam 1 Review Name:________________________________

1. Verify by substituion that every member of the family of functions $y(t) = \dfrac{1 + ce^t}{1 - ce^t}$ is a solution to the differential equation $y'(t) = \frac{1}{2}(y^2 - 1)$.

2. Construct a vector field for the differential equation $y' = 2x - xy$. Then sketch a few solution curves for different initial conditions, making sure to include the solution curve with initial condition $y(0) = 0$.



3. Find a general solution to the separable differential equation $\frac{dy}{dx} = \frac{\ln(x)}{xy}$ and then find the specific solution with initial condition $y(1) = 2$.

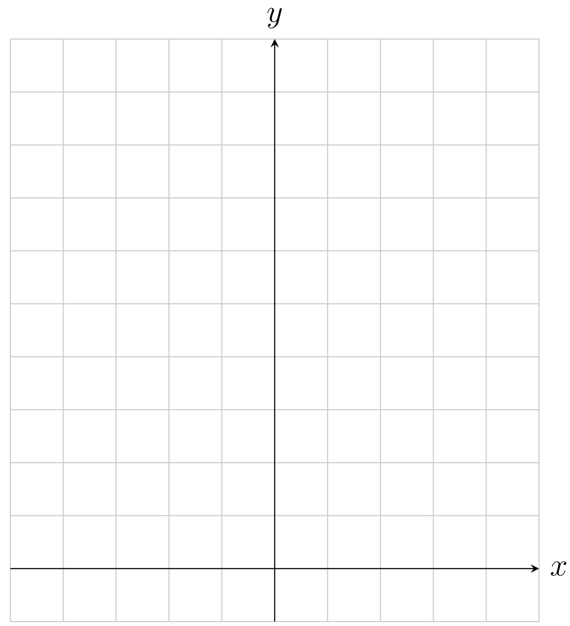
4. Suppose a cup of coffee is at 95°F in a 20°F room. Suppose further that, when the coffee is 70°F, the coffee is cooling at a rate of 1°F per minute. Set up a differential equation modeling this scenario, solve the differential equation in order to determine a function modeling the temperature of the coffee t-minutes from when it was 95°F. How long will it take for the coffee to reach 32°F?

5. A simple RC-circuit's charge on its capacitor can be modeled by the differential equation

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

where R is the resistance in ohms, Q is the charge in coulombs, C is the capacitance in farads, and E is the voltage. Suppose the resistor is 5 ohms, the capacitance is $0.5F$, and $E(t) = 60$ volts (constant). If the initial charge is $Q(0) = 0$ coulombs on the capacitor, use Euler's method to determine an approximation for the charge on the capacitor 0.5 seconds later using a step size of 0.1. Show your work nicely so I can follow it.

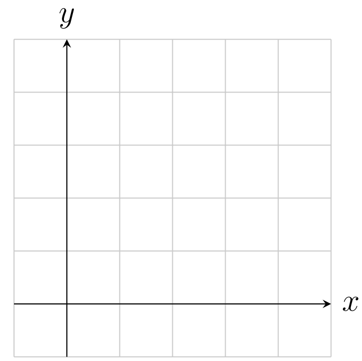
6. Given a population model of $\frac{dP}{dt} = kP\left(1 - \frac{P}{40}\right)\left(\frac{P}{60} - 1\right)$, draw a phase line next to a graph of specific solutions to the differential equation. What do the numbers 40 and 60 represent?



7. Given the differential equation $\frac{dy}{dt} = y(-y^2 + 4y + \alpha)$, draw a bifurcation diagram with several phase lines and cooresponding graphs of specific solutions.

8. Suppose a 100-gallon tank initially contains 100 gallons of sugar at a concentration of 0.25 lbs of sugar per gallon. Suppose that sugar is added to the tank at a rate of p pounds per minute, and that sugar water is removed after mixing at a rate of 1 gallon per minute. What value of p should we pick so that, when 5 gallons of sugar solution is left in the tank, the concentration is 0.5 pounds of sugar per gallon? Set up and solve a differential equation and show your work nicely.

9. Given the system of differential equations $\frac{dx}{dt} = -0.05x + 0.0001xy$ and $\frac{dy}{dt} = 0.1y - 0.005xy$, which models a predator/prey scenario, which variable represents the population of the prey and which the predator? Sketch a vector field and phase portrait for this scenario.



10. Given the linear system of differential equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{y}$$

show that any linera combination of the solutions $\mathbf{y}_1(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$ and $\mathbf{y}_2(t) = \begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$ are solutions to the differential system. What is the specific solution satisfying $\mathbf{y}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$?

11. Given the linear system of differential equations

$$\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix} \mathbf{y}$$

determine the fixed point(s) of the system and sketch a direction field and phase portrait for the system.