Math 256 Exam 2 Review - Sections 3.1 through 5.5

Name:______________________________

1. Solve the following initial value problems.

   a. $y'' - 6y' + 34y = 0$; $y(0) = 2$, $y'(0) = 1$

   b. $y^{(3)} + 12y'' + 36y' = 0$; $y(0) = 1$, $y'(0) = 5$, $y''(0) = -24$.

2. A mass, $m = 4$, is attached to both a spring with spring constant $k = 169$ and a dashpot with damping constant $c = 20$. If the motion is started with initial position $x_0 = 4$ and initial velocity $v_0 = 16$, find the position function $x(t)$ and determine whether the motion is overdamped, critically damped, or underdamped. If it is underdamped, write it in the form $x(t) = C_1 e^{-pt} \cos(\omega_1 t - \alpha_1)$.

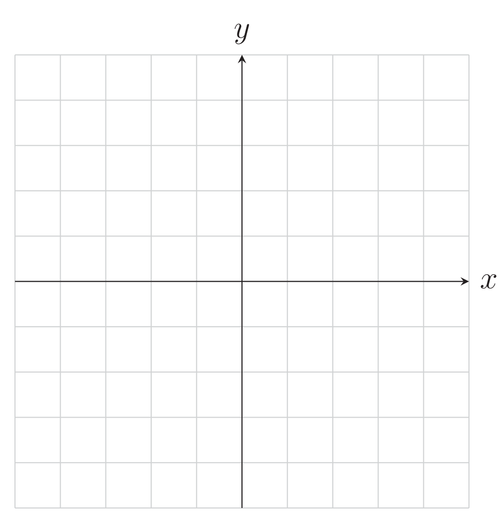
3. Find the general solution to the equation $y^{(4)} - 5y'' + 4y = e^x - xe^{2x}$

4. Given an RLC circuit with $R = 12\Omega$, $L = 2$ H, $C = 1/26$ F, attached to an alternating power source of 10 Volts with a frequency of $5/(2\pi)$, determine the charge on the capacitor, $Q(t)$, and current, $I(t)$, satisfying $I(0) = 0$ and $Q(0) = 0$.

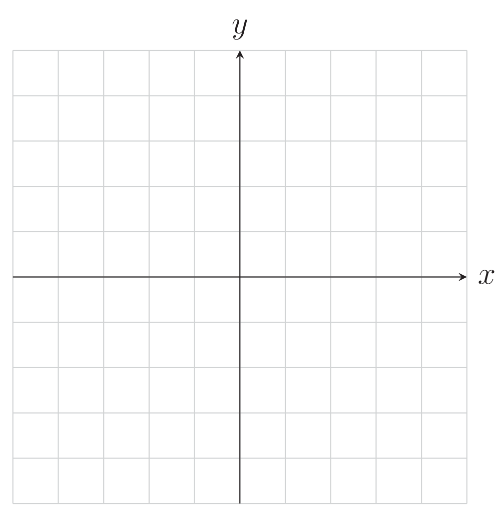
5. Find the general solution of the system $x' = 2x - 3y + 2\sin(2t)$, $y' = x - 2y - \cos(2t)$ via the elimination method.

6. Apply the eigenvalue method to find a general solution of the given systems.Draw a direction field and typical solution curves for each.

   a. $x_1' = 4x_1 + x_2$, $x_2' = 6x_1 - x_2$



b. $x_1' = 5x_1 - 9x_2$, $x_2' = 2x_1 - x_2$



7. The eigenvalues of the coefficient matrix can be found by inspection and factoring for the following system. Apply the eigenvalue method to find a general solution to the system $x_1' = 5x_1 + x_2 + 3x_3$, $x_2' = x_1 + 7x_2 + x_3$, $x_3' = 3x_1 + x_2 + 5x_3$.

8. Suppose you have two masses sitting in-line between two walls with springs connecting the left mass to the left wall, the left mass to the right mass, and the right mass to the right wall. The left mass is 1 unit, the right mass is 2 units. The spring constants are 2, 4, and 4 from left to right. Determine a stiffness matrix to set up a system of linear differential equations. Find the two natural frequencies of the system and describe its two natural modes of oscillation.

9. Find a general solution to $\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} \mathbf{x}$.

10. Find a general solution to $\mathbf{x}' = \begin{bmatrix} -19 & 12 & 84 \\ 0 & 5 & 0 \\ -8 & 4 & 33 \end{bmatrix} \mathbf{x}$.