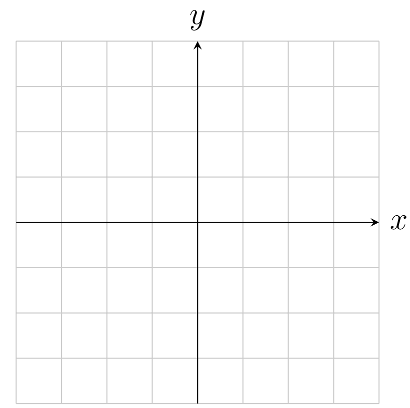
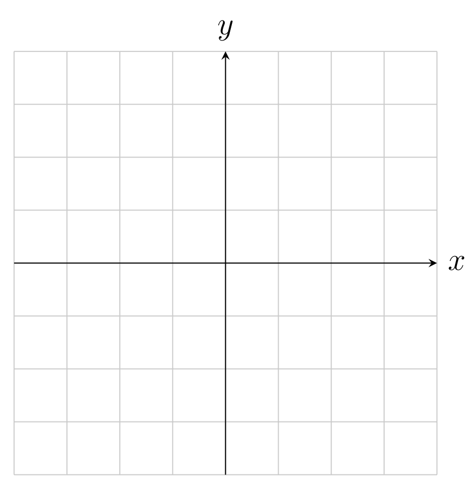
MTH 256 Exam 2 Review Eigenspaces through Forced Harmonic Oscillators
Name:______________________________

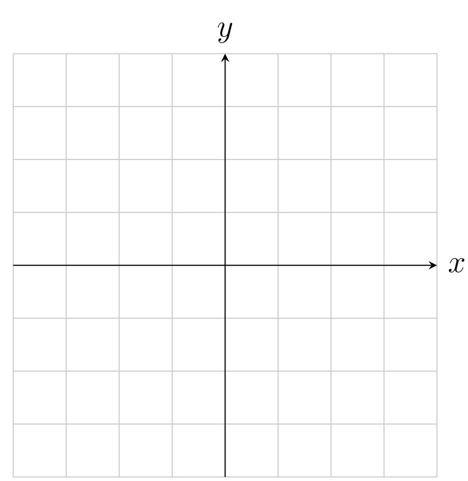
1. Given the system of differential equations $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 4 & -7 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, compute the eigenvalues and eigenvectors, sketch the direction field and stright-line solutions, and state the general solution to system. For each eigenvalue specify a corresponding straight-line solution. Find the particular solution with initial condition $(x(0), y(0)) = (-11, -1)$.



2. Given the system of differential equations $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -2 & -10 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, compute the eigenvalues and eigenvectors then use the real and imaginary parts to put together the general solution to the system of equations. Sketch the vector field and phase portrait. Determine the particular solution with initial condition $(x(0), y(0)) = (4, -4)$.



3. Find a general solution to $\frac{d\mathbf{y}}{dt} = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \mathbf{y}$. Sketch the vector field and phase portrait. Determine the particular solution with initial condition $(x(0), y(0)) = (-1, -2)$



4. The eigenvalues of the coefficient matrixf can be found by inspection and factoring for the following system. Apply the eigenvalue method to find a general solution to the system $x_1' = 5x_1 + 5x_2 + 2x_3$, $x_2' = -6x_1 + -6x_2 + -5x_3$, $x_3' = 6x_1 + 6x_2 + 5x_3$. Sketch the decoupled phase portraits separately.

5. Consider a harmonic oscillator with mass $m = 9$, spring constant $k = 4$, damping coefficient $c = 6$, with initial conditions $y(0) = 3$, $v(0) = 4$. Write the second-order differential equation and the corresponding first-order system, then find the general and specific solutions. Classify the oscillator and, if appropriate state the natural period and frequency. Sketch the phase portrait, including the solution curve for the given initial condition and sketch the $y(t)$ and $v(t)$ graphs of the solution with the given initial condition.

6. Suppose you have a mass-spring harmonic oscillator where $m = 3$, $c = 18$, and $k = 15$. However, an outside force is acting on the system via the function $F(t) = 2t + e^{-5t}$. Determine the particular solution satisfying $x(0) = 0$ and $x'(0) = 0$.

7. Consider an RLC circuit with $R = 200$ ohms Ω, $L = 5$ henry (H), and $C = 0.001$ farad (F). At time $t = 0$, when both $I(0)$ and $Q(0)$ are zero, the switch in the circuit is closed and an alternating current given by $E(t) = 100\sin(10t)$ is allowed into the circuit. Find the current in the cuircuit and the time lag of the steady periodic current behind the voltage.

8. Given the differential equation $\frac{d^2y}{dt^2} + 5y = 2\sin(2t)$, determine the general solution, the frequency of the beats, and the frequency of rapid oscillations. What is the particular solution for when $y(0) = 0$ and $y'(0) = 0$?