Name: Solutions

The first part of this review will be for the no-calculator portion of the exam. Meaning you should be sure that you are comfortable doing these problems with your calculator put away.

Exercise 1 Evaluate the following limits.

a. $\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{(x+4)} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}$

$= \lim_{x\to -4} \frac{(x^2+9)-25}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x\to -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x\to -4} \frac{(x+4)(x-4)}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5}$

$= \frac{-8}{\sqrt{16+9}+5} = \frac{-8}{10} = -4/5$

b. Use the squeeze theorem to show that $\lim_{x\to 0}(x^2\cos(20\pi x)) = 0$.

$-1 \le \cos(20\pi x) \le 1$

$-x^2 \le x^2\cos(20\pi x) \le x^2$

$\lim_{x\to 0} -x^2 \le \lim_{x\to 0} x^2\cos(20\pi x) \le \lim_{x\to 0} x^2$

$0 \le \lim_{x\to 0} x^2\cos(20\pi x) \le 0$

$\Rightarrow \lim_{x\to 0} x^2\cos(20\pi x) = 0$

c. $\lim_{x\to\infty}(e^{-2x}\cos x)$

$-1 \le \cos(x) \le 1$

$-\frac{1}{e^{2x}} \le \frac{\cos(x)}{e^{2x}} \le \frac{1}{e^{2x}}$

$\lim_{x\to\infty} -\frac{1}{e^{2x}} \le \lim_{x\to\infty} \frac{\cos(x)}{e^{2x}} \le \lim_{x\to\infty} \frac{1}{e^{2x}}$

$0 \le \lim_{x\to\infty} \frac{\cos(x)}{e^{2x}} \le 0$

$\Rightarrow \lim_{x\to\infty} \frac{\cos(x)}{e^{2x}} = 0$

d. $\lim_{t\to -\infty} \frac{t^2+2}{t^3+t^2-1}$

$= \lim_{t\to -\infty} \frac{t^2}{t^3}$

$= \lim_{t\to -\infty} \frac{1}{t}$

$= 0$

e. $\lim_{x\to 1} \frac{2-x}{(x-1)^2}$

$= \infty$

$\frac{2-1}{(\pm 0)^2} = \frac{1}{+0} = +\infty$

f. $\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2^-} \frac{x(x-2)}{(x-2)(x-2)}$

$= \lim_{x\to 2^-} \frac{x}{x-2}$

$= -\infty$

$\frac{2}{2^- - 2} = \frac{2}{-0} = -\infty$

Exercise 2 For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

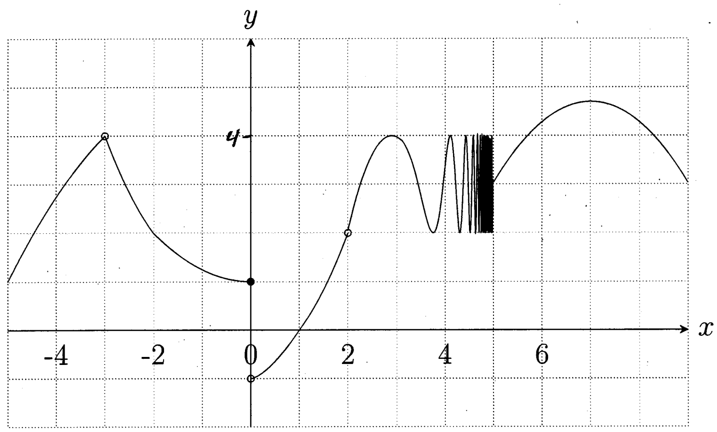


Figure 0.0.1: $y = h(x)$

a. $\lim_{x\to -3^-} h(x)$ = 4

b. $\lim_{x\to -3^+} h(x)$ = 4

c. $\lim_{x\to -3} h(x)$ = 4

d. $h(-3)$ is undefined

e. $\lim_{x\to 0^-} h(x)$ = 1

f. $\lim_{x\to 0^+} h(x)$ = -1

g. $\lim_{x\to 0} h(x)$ DNE

h. $h(0)$ = 1

i. $\lim_{x\to 2} h(x)$ = 2

j. $h(2)$ is undefined

k. $\lim_{x\to 5^+} h(x)$ = 3

l. $\lim_{x\to 5^-} h(x)$ DNE

Exercise 3 The graphs of f and g are given. Use them to evaluate each limit, if it exists. If it does not exist, explain why.

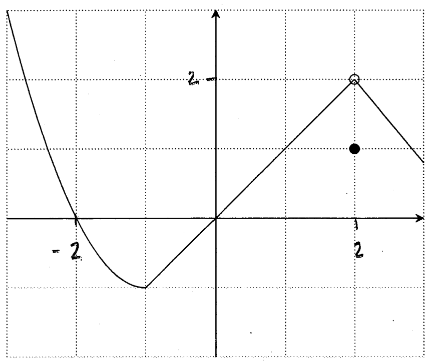


Figure 0.0.2: $y = f(x)$

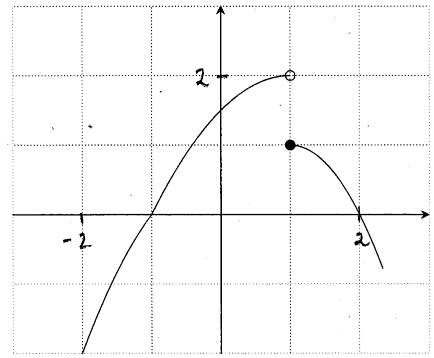


Figure 0.0.3: $y = g(x)$

a. $\lim_{x\to 2}[f(x)+g(x)] = 2+0$
$= 2$

b. $\lim_{x\to 1}[f(x)+g(x)]$ DNE

c. $\lim_{x\to 0}[f(x)g(x)] = 0 \cdot 1.5$
$= 0$

d. $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$ DNE

e. $\lim_{x\to 2}[x^3 f(x)] = 16$

f. $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+1} = 2$

Exercise 4 Draw a graph of a function m which satisfies all of the following properties.

a. The only discontinuities on m occur at -4 and 3 ✓

b. m has no x-intercepts ✓

c. $m(-6) = 5$ ✓

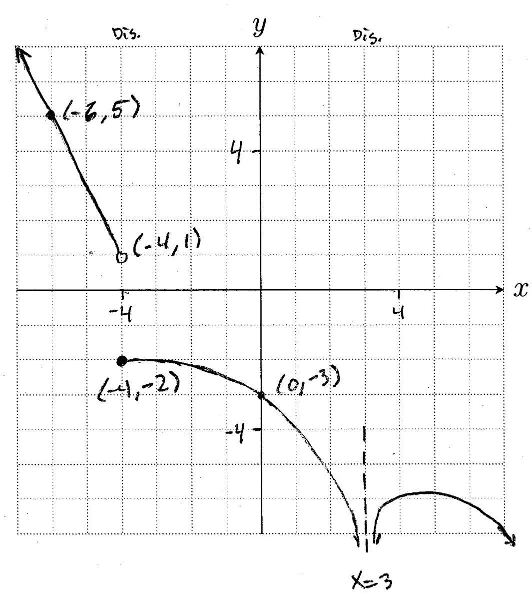
d. $\lim_{x\to -4^+} m(x) = -2$ ✓

e. $\lim_{x\to 3} m(x) = -\infty$ ✓

f. $\lim_{x\to \infty} m(x) = -\infty$ ✓

g. m has a constant slope of -2 over $(-\infty,-4)$ ✓

h. m is continuous over $[-4,3)$ ✓



Exercise 5 Let $f(x) = \begin{cases} \dfrac{4}{5-x} & \text{if } x < 1 \\ \dfrac{x-3}{x-3} & \text{if } 1 < x < 4 \\ 2x+1 & \text{if } 4 \le x \le 7 \\ \dfrac{15}{8-x} & \text{if } x > 7 \end{cases}$

a. Where is f discontinuous?

when $x = 1, 3, 4, 8$

holes

asymptote

$\to 1$ from left
$\to 9$ from right
$f(4) = 9$

b. Where is f continuous only from the left?

Nowhere

c. Where is f continuous only from the right?

when $x = 4$

d. Where does f have a removable discontinuity?

At $x = 1$ + $x = 3$

Exercise 6 Evaluate the following limits. Use substitution if necessary.

a. $\lim_{x\to 1} e^{x^2-x} = e^0 = 1$

b. $\lim_{x\to\pi} \sin(x + \sin(x)) = \sin(\pi + \sin(\pi))$
$= \sin(\pi)$
$= 0$

Let $f(t) = \dfrac{4t}{t+1}$.

a. What is the domain of f?

$D = \{t \mid t \neq -1\}$

b. Find $f'(t)$.

$$f'(t) = \lim_{h\to 0} \frac{\frac{4(t+h)}{t+h+1} - \frac{4t}{t+1}}{h}$$

$$= \lim_{h\to 0} \frac{4(t+h)(t+1) - 4t(t+h+1)}{(t+1)(t+h+1)h}$$

$$= \lim_{h\to 0} \frac{4(t^2+t+th+h) - 4t^2 - 4th - 4t}{(t+1)(t+h+1)h}$$

$$= \lim_{h\to 0} \frac{4h}{(t+1)(t+h+1)h} = \lim_{h\to 0} \frac{4}{(t+1)(t+h+1)}$$

$$= \frac{4}{(t+1)^2}$$

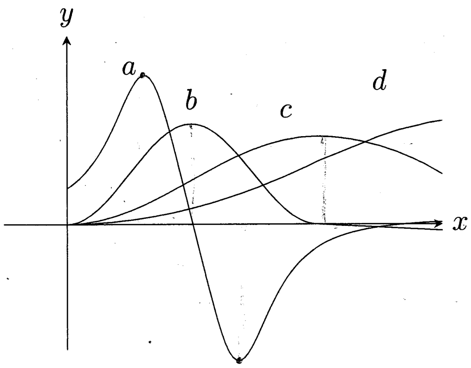
c. What is $f(1)$ and $f'(1)$?

$f(1) = 2 \quad f'(1) = 1$

d. What is the equation of the line which is tangent to f at $t = 1$?

$T_{f(1)}(t) = 1(t-1) + 2$
$= t + 1$

Exercise 7 The figure shows the graphs of f, f', f'', and f'''. Identify each curve and explain your choices.



a has slope $=0$ at places where no other function has an x-int so $a = f'''$

b has slope $=0$ when a has a x-int so $b' = a$ i.e. $b = f''$.

b has an xint when c has slope zero so $c = f'$

d is always increasing so its slope is always positive & c is always positive so $d = f$.

Exercise 8 Sketch the graph of a function that satisfies all of the given conditions.

√ i. $f'(x) > 0$ if $|x| < 2$ $\quad -2 < x < 2$

√ ii. $f'(x) < 0$ if $|x| > 2$ $\quad x < -2$ or $x > 2$

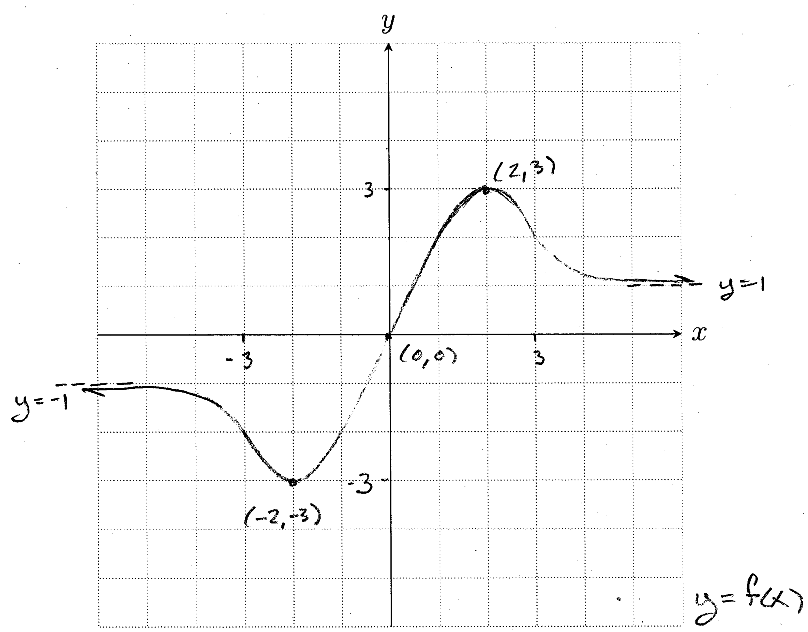
√ iii. $f'(2) = 0$

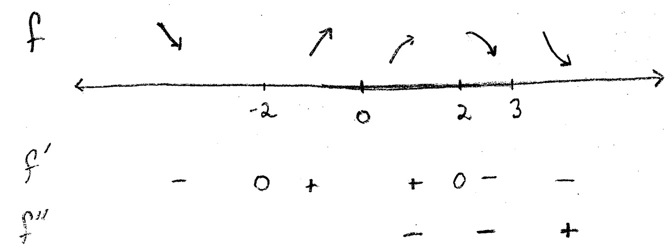
√ iv. $\lim_{x\to\infty} f(x) = 1$

√ v. $f(-x) = -f(x)$ ← Odd Function

√ vi. $f''(x) < 0$ if $0 < x < 3$

√ vii. $f''(x) > 0$ if $x > 3$





The second part of this review will be for the calculator allowed portion of the exam. You will be allowed any calculator you want but you shouldn't need anything more than a scientific calculator.

Exercise 9 The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2\sin(\pi t) + 3\cos(\pi t)$, where t is measured in seconds.

$s(1) = 2(0) + 3(-1) = -3$

a. Find the average velocity during each time period, organizing your work in a table. Be sure your table has proper headings that make sense in context.

i. $[1,2]$

ii. $[1,1.1]$

iii. $[1,1.01]$

iv. $[1,1.001]$

$m_{[1,b]} = \dfrac{s(b) - s(1)}{b-1} = \dfrac{s(b)+3}{b-1}$

$m_{[1,2]} = \dfrac{s(2)+3}{2-1} = \dfrac{3+3}{1} = 6$

$m_{[1,1.1]} = \dfrac{s(1.1)+3}{1.1-1} \approx \dfrac{-0.471}{0.1} = -4.71$

$m_{[1,1.01]} = \dfrac{s(1.01)+3}{1.01-1} \approx \dfrac{-0.0613}{0.01} = -6.13$

$m_{[1,1.001]} = \dfrac{s(1.001)+3}{1.001-1} \approx \dfrac{-0.00627}{0.001} = -6.27$

b. Estimate the instantaneous velocity of the particle when $t = 1$.

$s'(1) = -2\pi$ is what appears to be happening.

Exercise 10 If a rock is thrown upward on the planet Mars with a velocity of $10m/s$, its height (in meters) after t seconds is given by $H(t) = 10t - 1.86t^2$.

a. When will the rock hit the ground?

$0 = t(10 - 1.86t)$

$t = 10/1.86 = \frac{1000}{186} = \frac{500}{93} \approx 5.38$

The rock hits the ground about 5.38s after being thrown.

b. What is $v(t)$?

$v(t) = \lim_{h\to 0} \dfrac{10(t+h) - 1.86(t+h)^2 - (10t - 1.86t^2)}{h}$

$= \lim_{h\to 0} \dfrac{10t + 10h - 1.86(t^2 + 2th + h^2) - 10t + 1.86t^2}{h}$

$= \lim_{h\to 0} \dfrac{10h - 3.72th - 1.86h^2}{h}$

$= \lim_{h\to 0} 10 - 3.72t - 1.86h$

$= 10 - 3.72t$

c. What is the velocity of the ball when it hits the surface?

$v(500/93) = 10 - 3.72(500/93)$

$= -10$

The velocity of the rock upon impact is -10 m/s.