Name:______________________________

Exercise 1 Evaluate the following limits.

a. $\lim_{x\to\infty} \frac{\sin(x)}{x^2+x-1}$

b. $\lim_{t\to-\infty} \frac{3t^4+2t-1}{-5t^4+2t^2-1}$

c. $\lim_{x\to 3} \frac{x^2+2x-3}{x^2-6x+9}$

d. $\lim_{x\to\pi} e^{\sin^2(x)}$

Exercise 2 Let $f(x) = \begin{cases} \frac{(1-x)(x+2)}{x+2} & \text{if} \quad x \le -1 \\ \frac{1}{x} & \text{if} \quad -1 < x < 1 \\ x^2-2x & \text{if} \quad 1 \le x \end{cases}$. No need to show any work here.

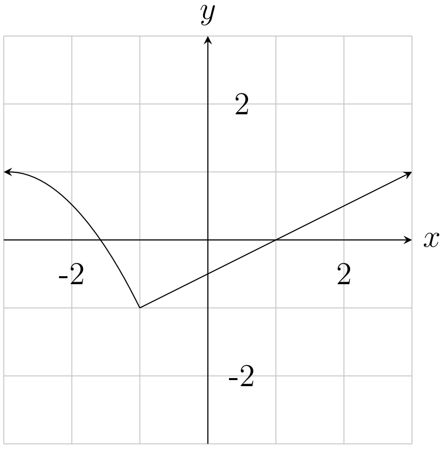
a. Where is f discontinuous?

b. Where is f continuous only from the left?

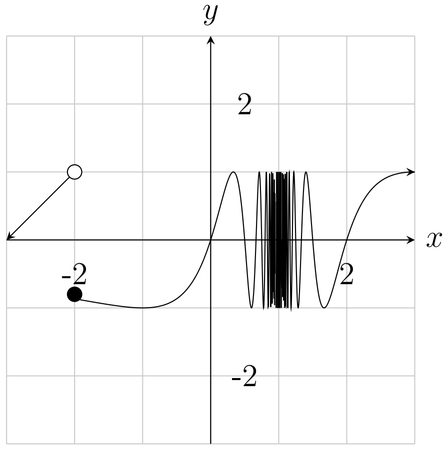
c. Where is f continuous only from the right?

d. Where does f have a removable discontinuity?

Exercise 3 The graphs of f and g are given. Use them to evaluate each limit, if it exists. If it does not exist, explain why.



$f(x)$



$g(x)$

a. $\lim_{x \to -1} [f(x) + g(x)]$

b. $\lim_{x \to 1} [f(x)g(x)]$

c. $\lim_{x \to 2} \frac{g(x)}{f(x)}$

d. $\lim_{x \to 3} \frac{f(x)}{x^2}$

Exercise 4 Let $f(t) = \frac{3t}{t-1}$.

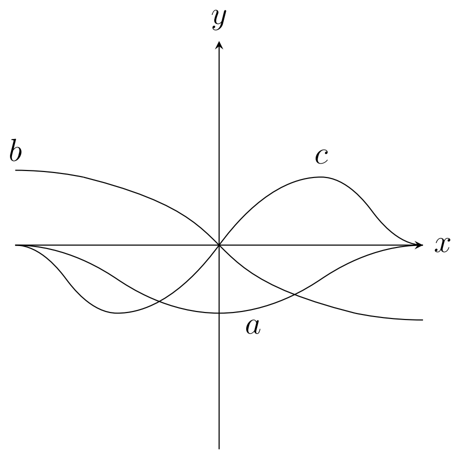
a. What is the domain of f?

b. Find $f'(t)$. State its domain.

c. What is $f(2)$ and $f'(2)$?

d. What is the equation of the line which is tangent to f at $t = 2$?

Exercise 5 The figure shows the graphs of f, f', and f''. Identify each curve and explain your choices.



Exercise 6 Sketch the graph of a function that satisfies all of the given conditions.

i. $f(0) = 2$

ii. The only discontinuities on f occur at -2 and 0.

iii. $\lim_{x \to -2^-} f(x) = -\infty$

iv. $\lim_{x \to -2^+} f(x) = \infty$

v. $f'(x) > 0$ if $x > 0$

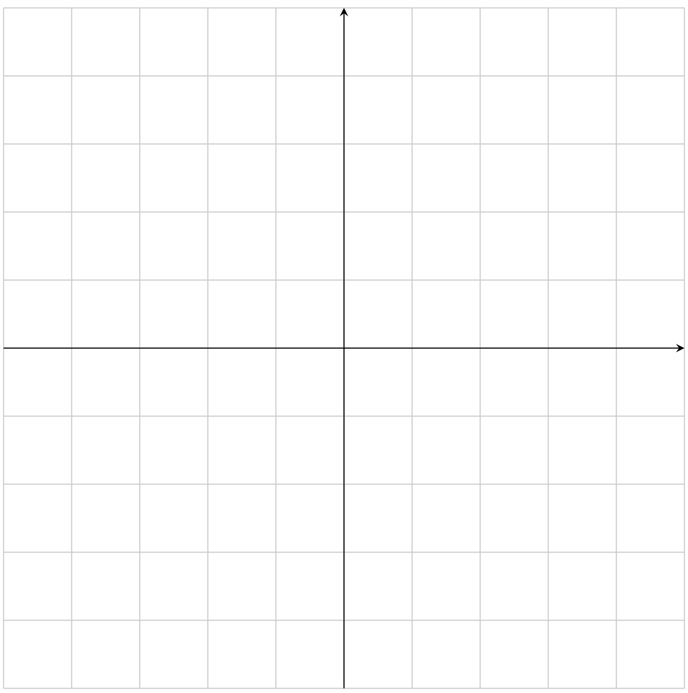
vi. $f'(x) < 0$ if $x < -2$ or $-2 < x < 0$

vii. $f''(x) < 0$ if $x < -2$ or $x > 3$

viii. $f''(x) > 0$ if $-2 < x < 0$ or $0 < x < 3$

ix. $\lim_{x \to \infty} f(x) = 2$

x. $\lim_{x \to -\infty} f(x) = -1$



Exercise 7 Find an equation of the tangent line to the curve at the given location:

a. $y = \dfrac{2x}{x+1}$ at (1,1)

b. $\sin(x+y) = 2x - 2y$ at (π, π)

c. $y = 3\arccos\left(\dfrac{x}{2}\right)$ at $(1, \pi)$

d. $y = \ln\left(xe^{x^2}\right)$ at (1,1)

Exercise 8 On what interval is the function $f(x) = 5x - e^x$ increasing?

Exercise 9 Find the second derivative of $f(x) = x^4e^x$

Exercise 10 Find all points on the graph of the function $f(x) = 2\sin(x)$ at which the tangent line is horizontal. Describe them using set notation.

Exercise 11 A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

a. If $h(x) = f(g(x))$, find $h'(1)$

b. If $H(x) = g(f(x))$, find $H'(1)$

Exercise 12 Find an equation of the tangent line to the parametric curve $x = t^4 + 1$, $y = t^3 + t$ when $t = -1$.

Exercise 13 Find y'' of $9x^2 + y^2 = 9$ by implicit differentiation.

Exercise 14 Use logarithmic differentiation to find the derivative of the function $f(x) = x^{\cos(x)}$.

The following part of the exam will be in the calculator portion so feel free to practice with your calculator at hand.

Exercise 15 The displacement (in meters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 4\sin(\pi(t - .5))$, where t is measured in seconds.

a. Find the average velocity during each time period, organizing your work in a table. Be sure your table has proper headings that make sense in context.

   i. $[0, 0.1]$
   ii. $[0, 0.01]$
   iii. $[0, 0.001]$
   iv. $[0, -0.1]$
   v. $[0, -0.01]$
   vi. $[0, -0.001]$

b. Use the table to estimate the instantaneous velocity of the particle when $t = 0$.

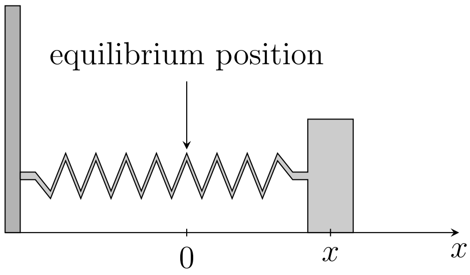
Exercise 16 Suppose air is being pumped into a balloon such that the volume of the balloon, given in centiliters, can be determined by the function $V(t) = -0.4t^2 + 6t$ where t is measured in seconds.

a. Suppose the balloon will pop when the volume hits 22.5 centiliters in volume. How many seconds will this take?

b. Determine the function $V'(t)$ and describe what it means in this context. Include units in your description. You do not need to show your work.

c. What is the rate at which the volume of the balloon is expanding when the balloon pops?

Exercise 17 A mass on a spring vibrates horizontally on a smooth level surface (see figure). Its equation of motion is $x(t) = 8\sin(t)$, where t is in seconds and x in centimeters.



a. Find the velocity and acceleration at time t.

b. Find the position, velocity, and acceleration of the mass at time $t = \dfrac{2\pi}{3}$. In what direction is it moving at that time?

Exercise 18 Suppose you have a sphere whose surface area S is a function of its radius r. Further suppose its radius is a function of its volume V and $r(64) = 2$.

If $\left.\dfrac{dS}{dr}\right|_{r=2\text{cm}} \approx 50.265\dfrac{\text{cm}^2}{\text{cm}}$

and $\left.\dfrac{dr}{dV}\right|_{V=64\text{cm}^3} \approx 0.0129\dfrac{\text{cm}}{\text{cm}^3}$

State the value and meaning of $\left.\dfrac{dS}{dV}\right|_{V=64\text{cm}^3}$ including units.