Math 256 Final Exam Review (1.1 - 7.6) - Part I No Calculator

Name:________________________________

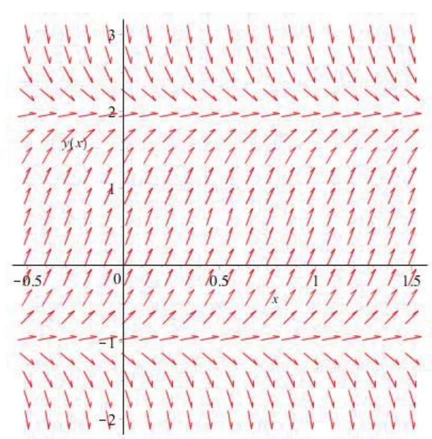
1. Find the particular solution of the initial value problems.

   a. $\frac{dy}{dx} = y\cot(x), \quad y(\pi/2) = \frac{\pi}{2}$

   b. $x\frac{dy}{dx} = 3y + x^4\cos(x), \quad y(2\pi) = 0$

2. Solve the differential equation $xy' = y + 2\sqrt{xy}$ using the substitution $\nu = \frac{y}{x}$.

3. Solve the initial value problem $\frac{dx}{dt} = 3(x+1)(2-x)$, $x(0) = 0$. Then sketch the graphs of several solutions of the given differential equation and highlight the indicated particular solution.



Name:______________________________

4. Suppose that a motorboat is moving at 40 ft/s when its motor suddenly quits, and that 10 s later the boat has slowed to 20 ft/s. Assume that the resistance it encounters while coasting is proportional to its velocity. How far will the boat coast in all?

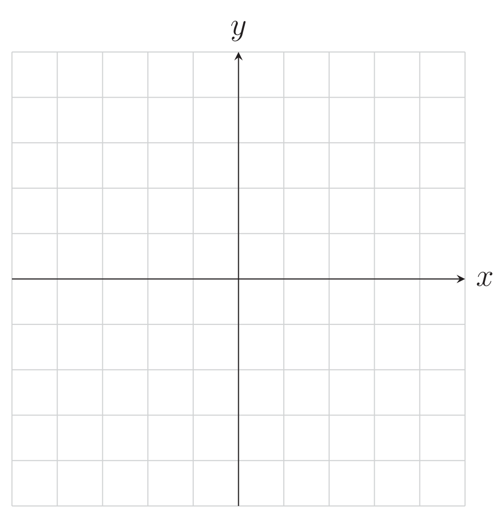
5. Apply Euler's method to approximate the solution to $y' = 3y - x$ with initial value $y(0) = 1$ over the interval $[0, .5]$ with step size $h = 0.1$. Is your approximate solution an overestimate or underestimate?

6. Given an RLC circuit with $R = 200\Omega$, $L = 5$ H, $C = 0.001$ F, attached to an alternating power source given by $E(t) = 100\sin(10t)$ V, determine the general solutions for the charge on the capacitor, $Q(t)$, and current, $I(t)$.

7. Apply the eigenvalue method to find a general solution of the given system. Draw a direction field and typical solution curves.

   This may or may not be helpful:
   $\mathbf{x(t)} = c_1 e^{pt}(\mathbf{a}\cos(qt) - \mathbf{b}\sin(qt)) + c_2 e^{pt}(\mathbf{b}\cos(qt) + \mathbf{a}\sin(qt))$

   $x_1' = -3x_1 + 4x_2,\ x_2' = 6x_1 - 5x_2$



8. Suppose you have two masses sitting in-line between two walls with springs connecting the left mass to the left wall, the left mass to the right mass, and the right mass to the right wall. Both masses are 1 unit. The spring constants are 1, 4, and 1 from left to right. Determine a stiffness matrix to set up a system of linear differential equations. Find the two natural frequencies of the system and describe its two natural modes of oscillation.

9. Find a general solution to $\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix} \mathbf{x}$.

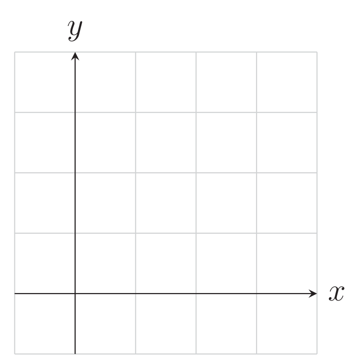
10. Given the system of differential equations $\frac{dx}{dt} = 30x - 2x^2 - xy$, $\frac{dy}{dt} = 80y - 4y^2 + 2xy$ which models the rates of changes of two interacting species populations, describe the type of x- and y-populations involved (exponential or logistic) and the nature of their interaction - competition, cooperation, or predation. Then find and characterize the system's critical points (type and stability). Determine what nonzero x- and y-populations can coexist. Then construct a phase plane portrait that enables you to describe the long-term behavior of the two populations.

11. Use Laplace transforms to solve the initial value problem $x'' + 8x' + 10x = t$; $x(0) = x'(0) = 0$.

12. Consider a mass-spring-dashpot system with $m = 1$, $c = 6$, and $k = 6$ in appropriate units. Suppose that the system is acted upon by a force

$$f(t) = \begin{cases} t & \text{if } 0 \leq t < 3 \\ 0 & \text{if } t \geq 3 \end{cases}$$

and that $x(0) = x'(0) = 0$.
Solve the initial-value problem $mx'' + cx' + kx = f(t)$ and graph the position function $x(t)$.



13. A mass $m = 2$ is attached to a spring with constant $k = 6$; there is no dashpot. The mass is released from rest with $x(0) = 2$. At the instant $t = \pi$, the mass is struck with a hammer, providing an impulse $p = 6$. Determine the motion of the mass.