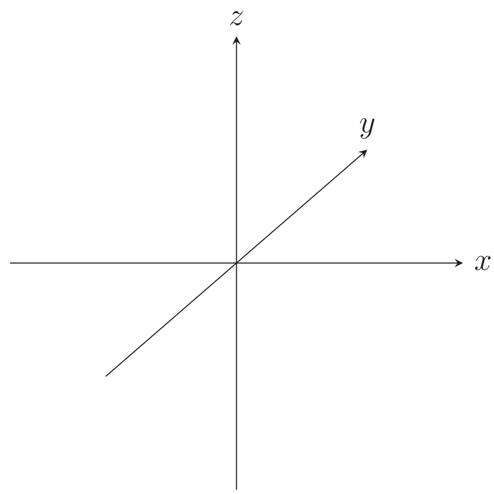
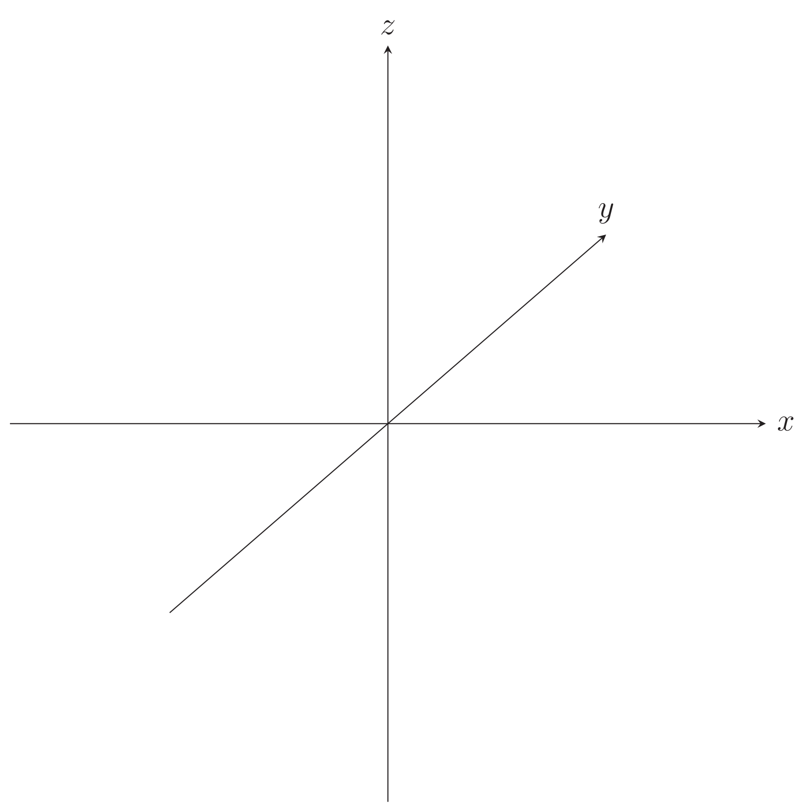
Math 253 Final Review Name: ___________________

1. Describe the set of points which satisfy the equation $x^2 + y^2 = 1$ in $\mathbb{R}^3$.



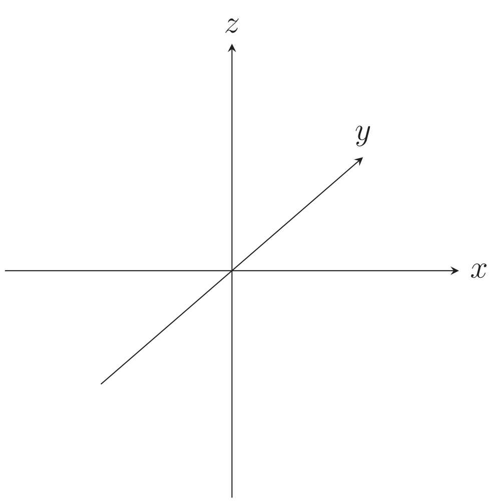
2. What are the projections of the point $(-2, 4, -5)$ on the $xy-$, $yz-$ and $xz-$planes? Draw a rectangular box with the origin and $(-2, 4, -5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.



3. Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere, and find its center and radius.

4. What region in $\mathbb{R}^3$ is represented by the inequalities

$$1 \leq x^2 + y^2 + z^2 \leq 4 \text{ and } z \leq 0?$$

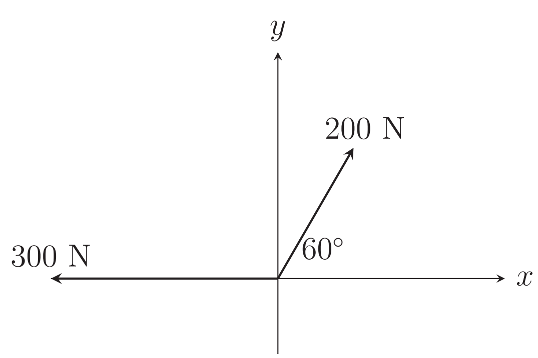


5. Find an equation of the set of all points equidistant from the points $A = (-2, 3, 3)$ and $B = (1, -3, 6)$.

6. Write an inequality to describe the solid upper hemisphere of the sphere of radius 2 centered around the point (1,1,1).

7. If $\mathbf{v}$ lies in the 2nd quadrant and makes an angle of $\dfrac{\pi}{6}$ with the positive y-axis and $|\mathbf{v}| = 3$, find $\mathbf{v}$ in component form. What angle does $\mathbf{v}$ make with the positive x-axis?

8. Find the magnitude of the resultant force and the angle it makes with the positive x-axis.

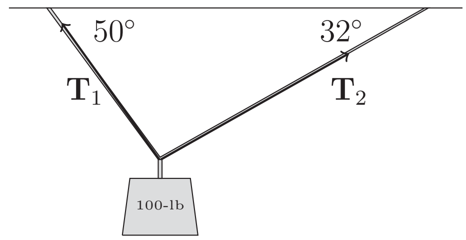


9. Determine the angle between the following vectors.

a. $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

b. $\mathbf{a} = \langle 2, 2, -1 \rangle$, $\mathbf{b} = \langle 5, -3, 2 \rangle$.

10. A 100-lb weight hangs from two wires as shown in the figure below. Find the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and the magnitudes of the tensions.



11. A force is given by a vector $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and moves a particle from the point $P = (2, 1, 0)$ to the point $Q = (4, 6, 2)$. Find the work done.

12. Determine the projection of $\mathbf{b}$ onto $\mathbf{a}$ for the given vectors and then determine $\text{orth}_{\mathbf{a}}\mathbf{b}$ for each.

a. $\mathbf{b} = \langle 1, 1, 2, \rangle$, $\mathbf{a} = \langle -2, 3, 1 \rangle$

b. $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

13. A bolt is tightened by applying a 40-N force to a 0.25-m wrench at an angle of 75° with the arm of the wrench. Find the magnitude of the torque about the center of the bolt.

14. Find the area of the parallelogram with vertices $K = (1, 2, 3)$, $L = (1, 3, 6)$, $M = (3, 8, 6)$, and $N = (3, 7, 3)$.

15. Find a nonzero vector orthogonal to the plane through the points $P = (-1, 3, 1)$, $Q = (0, 5, 2)$, and $R = (4, 3, -1)$ and then find the area of the triangle PQR.

16. Determine the volume of the parallelepiped, described by the vectors $\mathbf{a} = -4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}$, and $\mathbf{c} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

17. Suppose that $\mathbf{a} \neq \mathbf{0}$.

    a. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

    b. If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

    c. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

18. Determine whether each statement is true or false.

    a. Two lines parallel to a third line are parallel.

    b. Two lines perpendicular to a third line are parallel.

    c. Two planes parallel to a third plane are parallel.

    d. Two planes perpendicular to a third plane are parallel.

    e. Two lines parallel to a plane are parallel.

    f. Two lines perpendicular to a plane are parallel.

    g. Two planes parallel to a line are parallel.

    h. Two planes perpendicular to a line are parallel.

    i. Two planes either intersect or are parallel.

    j. Two lines either intersect or are parallel.

    k. A plane and a line either intersect or are parallel.

19. Find a vector equation and parametric equations for the line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t$, $y = 6 - 3t$, $z = 3 + 9t$.

20. Find vector, parametric, and symmetric equations of the line that passes through the points $A = (2, 4, -3)$ and $B = (3, -1, 1)$. At what point does the line intersect the xy-plane?

21. Find parametric equations and symmetric equations for the line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

22. Determine whether the lines L_1 and L_2 described below are parallel, skew, or intersecting. If they intersect, find the point of intersection.

a.

L_1: $x = -6t,\ y = 1 + 9t,\ z = -3t$
L_2: $x = 1 + 2s,\ y = 4 - 3s,\ z = s$

b.

$$L_1\colon\ \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$$
$$L_2\colon\ \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$$

23. Find an equation of the plane that contains the line $x = 1 + t,\ y = 2 - t,\ z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$.

24. a. Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

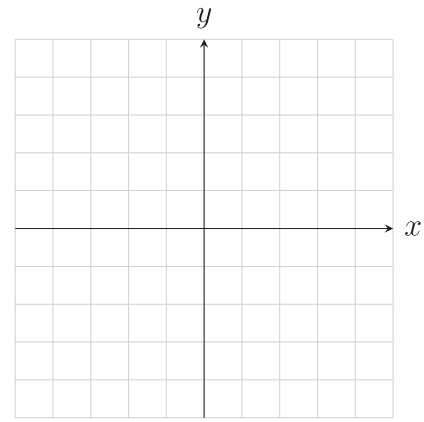
b. Find symmetric equations for the line of intersection L of these two planes.

25. Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$.

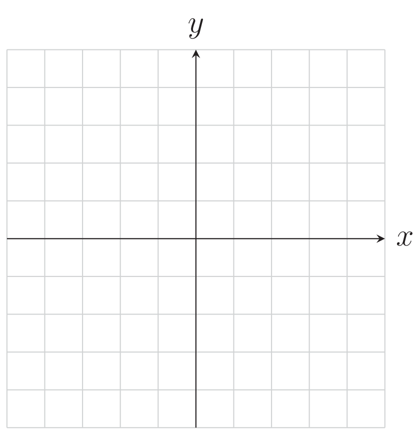
26. Find the distance between the skew lines described by the parametric equations

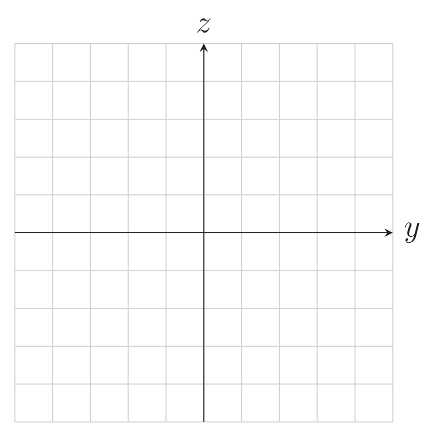
$$\begin{array}{llll} L_1: & x = 1 + t & y = -2 + 3t & z = 4 - t \\ L_2: & x = 2s & y = 3 + s & z = -3 + 4s \end{array}$$

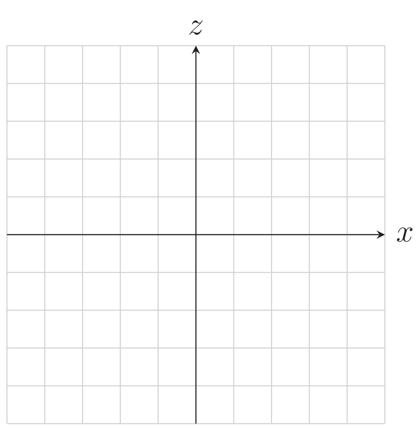
27. Find and sketch the domain of $f(x, y) = \ln(x^2 + y^2 - 2)$.

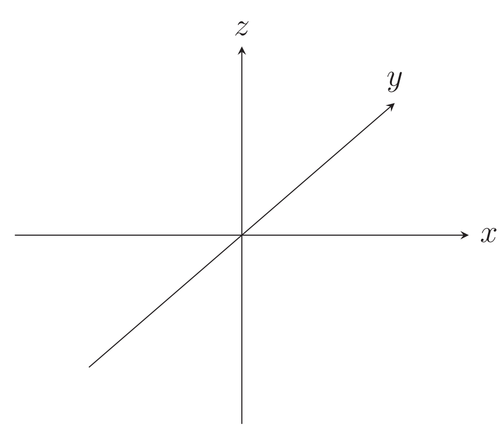


28. Use traces to sketch the graph of $f(x, y) = x^2 - y^2$





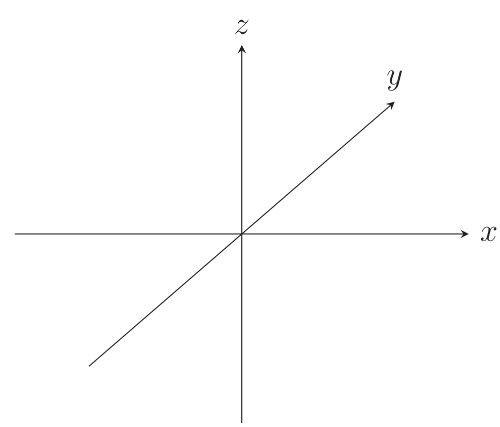




29. Classify $x^2 + 2z^2 - 6x - y + 10 = 0$ from one of the standard forms shown in chapter 9.6.

30. Plot the points whose coordinates are given. Then find the rectangular coordinates of the point.

a. $\left(3, \frac{3\pi}{4}, -2\right)_C$



b. $\left(2, \frac{\pi}{2}, \frac{\pi}{4}\right)_S$

31. Change the point $(1, -1, 4)$ from rectangular to cylindrical coordinates and spherical coordinates.

32. Describe in words the surface whose equation is given by $\phi = 5$.

33. Identify the surface whose equation is given.

a. $z = 4 - r^2$

b. $\rho^2(\sin^2 \phi \sin^2 \theta + \cos^2 \phi) = 9$

34. Write the equation in (a) cylindrical coordinates and (b) spherical coordinates.

$x^2 - y^2 - z^2 = 1$

35. Determine whether the following series converge or diverge and **justify** your answer.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n) + 1}{4^n}$

c. $\displaystyle\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{-2}{\sqrt{1+n^4}}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n n!}{(2n)!}$

36. Determine the radius and interval of convergent for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{\sqrt{n^2+1}}$$

37. For the following function, approximate f by a Taylor polynomial with degree n at the number a. Then use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

$$f(x) = e^x,\ a = -1,\ n = 2,\ -1.2 \leq x \leq -0.8$$