Math 253 FWH 4 - Geometric Series Applications Name:_____

1. Take the continuous number line from 0 to 1 and remove the middle section of length $\frac{1}{4}$. You then have intervals $[0, 3/8]$ and $[5/8, 1]$ of length $3/8$ each. Now remove a length of $\frac{1}{4^2}$ from the middle of these remaining intervals and determine the length of the remaining 4 intervals. Repeating this process so that at the third stage you remove lengths of $\frac{1}{4^3}$ from the middle of the remaining 4 intervals and so on and so forth. Determine the length of the intervals remaining after the n^{th} stage and the total length of the number line remaining. Then show that if this process is repeated an infinite number of times that the total length of the number line remaining will be $1/2$. Why is this a fascinating fact? Note: Do this on scratch paper and show me the pertinent work. I do not want to see your scratch work as you figure this out. I do want to see drawings of the lines for the first few steps. Hint, you're building a geometric series!

2. In the figure there are infinitely many circles inscribed in a 45° right triangle as shown below. Find the total area occupied by the circles if the lengths of the sides of the triangle are 1.