Name:________________________________

15.1 Distance Traveled

Exercise 15.1.1 The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

a. Find the velocity at time t.

b. What is the velocity after 2 seconds?

c. When is the particle at rest?

d. When is the particle moving forward?

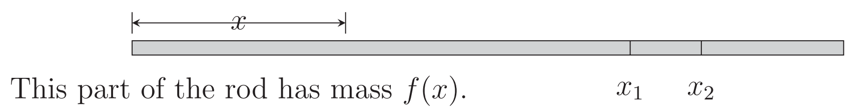
e. Draw a diagram to represent the motion of the particle.

f. Find the total distance traveled by the particle during the first five seconds.

g. Find the acceleration at time t and after 4 seconds.

15.2 Linear Density

Exercise 15.2.1 If a rod or piece of wire is homogeneous, then its linear density is uniform and is defined as the mass per unit length ($\rho = m/l$). Suppose, however, that the rod is not homogeneous but that its mass measured from its left end to a point x is $m = f(x)$, as shown in the below figure.



Suppose that $m = f(x) = \sqrt{x}$ is measured in kilograms and that x is measured in meters.

a. Determine the average density of the part of the rod given by $1 \leq x \leq 1.2$.

b. Determine the density right at $x = 1$.

15.3 Isothermal Compressibility

Exercise 15.3.1 One of the quantities of interest in thermodynamics is compressibility. If a given substance is kept at a constant temperature, then its volume V depends on its pressure P. We can consider the rate of change of volume with respect to pressure - namely, the derivative $\frac{dV}{dP}$. As P increases, V decreases, so $\frac{dV}{dP} < 0$. The **compressibility** is defined by introducing a minus sign and dividing this derivative by the volume V:

$$\text{isothermal compressibility} = \beta = -\frac{1}{V}\frac{dV}{dP}$$

Thus β measures how fast, per unit volume, the volume of a substance decreases as the pressure on it increases at constant temperature.

a. Suppose the volume V (in cubic meters) of a sample of air at 25°C was found to be related to the pressure P (in kilopascals) by the equation

$$V = \frac{5.3}{P}.$$

Find the rate of change of V with respect to P when $P = 50$ kPa.

b. Determine the compressibility at that pressure.

15.4 Populations Growth

Exercise 15.4.1 Let $n = f(t)$ be the number of individuals in an animal or plant population at time t. The change in the population size between the times $t = t_1$ and $t = t_2$ is $\Delta n = f(t_2) - f(t_1)$, and so the average rate of growth during the time period $t_1 \leq t \leq t_2$ is

$$\text{average rate of growth} = \frac{\Delta n}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

The **instantaneous rate of growth** is obtained from this average rate of growth by letting the time period Δt approach 0:

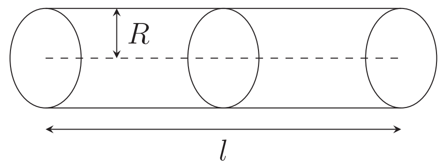
$$\text{growth rate} = \lim_{\Delta t \to 0} \frac{\Delta n}{\Delta t} = \frac{dn}{dt}$$

a. Suppose that by sampling the population of a bacteria at certain intervals it is determined that the population doubles every hour. If the initial population is n_0, determine the population as a function of time.

b. If we start with an initial population of $n_0 = 100$ bacteria, determine the growth rate after 4 hours.

15.5 Fluid Dynamics

Exercise 15.5.1 When we consider the flow of blood through a blood vessel, such as a vein or artery, we can model the shape of the blood vessel by a cylindrical tube with radius R and length l.



Because of friction at the walls of the tube, the velocity v of the blood is greatest along the central axis of the tube and decreases as the distance r from the axis increases until v becomes 0 at the wall. The relation ship between v and r is given by the **law of laminar flow** discovered by the French physician Jean-Louis-Marie Poiseuille in 1840. This law states that

$$v = \frac{P}{4\eta l}(R^2 - r^2)$$

where η is the viscosity of the blood and P is the pressure difference between the ends of the tube. If P and l are constant, then v is a function of r with domain $[0, R]$.

a. Determine $\frac{dv}{dr}$

b. For one of the smaller human arteries we can take $\eta = 0.027$, $R = 0.008$cm, $l = 2$ cm, and $P = 4000$ dynes/cm^2. Determine $v(r)$ for these conditions.

c. What is the speed of the blood at $r = 0.002$ cm?

d. What is the instantaneous change of velocity with respect to r when $r = 0.002$?