Math 253 GWS 10 - 8.8 Taylor's Inequality and Applications Name:________________________

1. Given the following functions, determine a value, M, such that $|f^{(n+1)}(x)| < M$ for the given value of n and the given interval.

   (a) $f(x) = x^{-2}, \quad n = 2, \quad 0.9 \le x \le 1.1$

   (b) $f(x) = \ln(1+2x), \quad n = 3, \quad 0.5 \le x \le 1.5$

2. Approximate $f(x) = x^{2/3}$ with T_3 about $a = 1$. Then use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when x lies in the interval $[0.8, 1.2]$. Check your results by graphing $|R_3(x)|$

3. Use Taylor's Inequality to estimate the range of values of x for which $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ is accurate within $|error| < 0.005$.

4. The period of a pendulum with length L that makes a maximum angle of θ_0 with the vertical is

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2(x)}}$$

where $k = \sin\left(\frac{1}{2}\theta_0\right)$ and g is the acceleration due to gravity.

a. Expand the integrand as a binomial series to show that

$$T = 2\pi\sqrt{\frac{L}{g}} \left[1 + \frac{1^2}{2^2}k^2 + \frac{1^2 3^2}{2^2 4^2}k^4 + \frac{1^2 3^2 5^2}{2^2 4^2 6^2}k^6\right]$$

Hint: You'll need to use the fact that $\int_0^{\pi/2} \sin^n(x)dx = \dfrac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot n} \cdot \dfrac{\pi}{2}$ when n is even.

Note that this gives $T \approx 2\pi\sqrt{L/g}$ and a better approximation is $T \approx 2\pi\sqrt{L/g}(1 + \frac{1}{4}k^2)$. Both of these approximations are seen in physics textbooks.