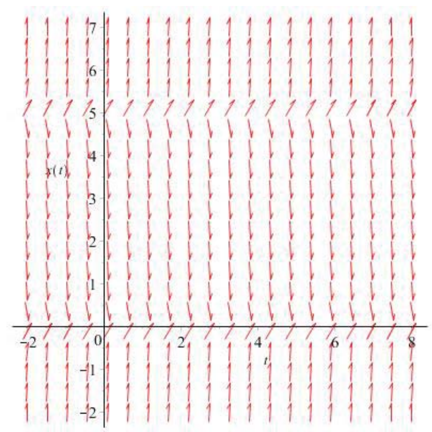
Math 256 GWS 4 - 2.1 and 2.2 Population Models and Equilibrium Solutions

Name:______________________________

1. Solve the initial value problem $\frac{dx}{dt} = 3x(x-5)$, $x(0) = 2$. Then sketch the graphs of several solutions of the given differential equation and highlight the indicated particular solution.

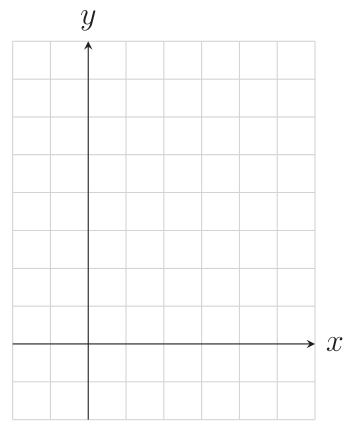


2. The time rate of change of an alligator population P in a swamp is proportional to the square of P. The swamp contained a dozen alligators in 1988, two dozen in 1998. When will there be four dozen alligators in the swamp? What is the population tending to as time goes forward?

3. Consider a population $P(t)$ satisfying the extinction-explosion equation $\frac{dP}{dt} = aP^2 - bP$, where $B = aP^2$ is the time rate at which births occur and $D = bP$ is the rate at which deaths occur. If the initial population is $P(0) = P_0$ and B_0 births per month and D_0 deaths per month are occuring at time $t = 0$, show that the threshold population is $M = D_0P_0/B_0$.

4. Given $\frac{dx}{dt} = f(x)$, first solve the equation $f(x) = 0$ to find the critical points of the given autonomous differential equation. Then analyze the sign of $f(x)$ to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the differential equation. Next, solve the differential equation explicitly for $x(t)$ in terms of t. Finally, sketch typical solution curves for the given differential equation and verify visually the stability of each critical point.

   a. $\frac{dx}{dt} = x^2 - 4x$



   b. $\frac{dx}{dt} = x^2 - 5x + 4$