6.1 The Derivative Function and Tangent Lines

Definition 6.1.1

The word **Derivative** is a synonym for slope in the following context: Given a function, f, we say "The Derivative of f at a is ____________." In math symbols, we write $f'(a) =$ ______. This means that "the slope of f at a is ____________." We may calculate the derivative of f at a using

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

We often read $f'(a)$ as "f-prime of a."

Note: Noah is not a big fan of this. I'll explain after the following definition.

Definition 6.1.2

The word **Derivative** is also used to ask for a **Derivative Function**, as in "What is the derivative of f?" In this case we are not mentioning at a specific x-value, and thus are requesting for the function that will give the slope for *any* x-value.

Given a function, f, its **Derivative Function**, denoted $f'(x)$ or $\frac{df}{dx}$, is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

That is, the derivative function will output the **slope** of f at any given x-value.
We usually read $f'(x)$ as "f-prime of x."

Note Follow Up: The reason I don't like the first definition of a derivative at a given x-value, is because it is just as easy to find the general derivative function and then plug the given x-value in after. This then allows us to plug in *any* x-value rather than only having the derivative for the specific value.

Example 6.1.1 Given the function $f(x) = x^2 - 8x + 9$:

a. Use the definition of derivative (the limit as $h \to 0$ of the difference quotient) to find $f'(x)$.

b. Find equation of the tangent line to $f(x)$ when $x = 3$. That is, find $T_{f(3)}(x)$.

Example 6.1.2 Given the function $f(x) = \dfrac{5}{x-2}$:

a. Use the definition of derivative (the limit as $h \to 0$ of the difference quotient) to find $f'(x)$.

b. Find equation of the tangent line to $f(x)$ when $x = 3$. That is, find $T_{f(3)}(x)$.

6.2 Applications of The Derivative Function

Definition 6.2.1

The **Instantaneous Rate of Change** of a function at a specific instance, is an expression used when a function has an input and output defined by a certain real-world scenario with units. It is the derivative of the function at the given instance, with units of the output unit of the original function divided by the input unit of the original function.

Definition 6.2.2

The **Instantaneous Velocity** of an object is equal to the derivative of the position of the object with respect to time. Since the derivative of position will have units of $\frac{length}{time}$ (the output of position unit over the input unit for the position function), we see this matches up with our preconceived notion of velocity.

Example 6.2.1 Suppose that a ball is dropped from the upper observation deck of the CN Tower, 450 meters above the ground. Use the formula

$$h(t) = -4.9t^2 + v_0 t + h_0$$

which outputs the height of an object, in meters, t seconds into its flight when its initial upward velocity is v_0 in meters per second and its initial height is h_0 in meters, to answer the following questions.

a. What is the velocity of the ball 5 seconds into its fall?

b. How fast is the ball traveling when it hits the ground?