9.1 The Power Rule

Example 9.1.1 Suppose c is any real number. What is $\frac{d}{dx}(c)$?

Example 9.1.2 Suppose n is a positive integer. Show that $\frac{d}{dx}(x^n) = nx^{n-1}$.

Example 9.1.3 Find the derivative functions, using the power rule proven above, of the following functions.

a. $f(x) = x^6$

b. $\frac{d}{dr}(r^3)$

Theorem 9.1.1

General power rule: If n is any real number, then $\frac{d}{dx}(x^n) = nx^{n-1}$

Example 9.1.4 Differentiate the following functions.

a. $f(x) = \frac{1}{x^2}$

b. $y = \sqrt[3]{x^2}$

9.2 Linearity of Differentiation

Example 9.2.1 Show that $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x) = c \cdot f'(x)$ for any real number c and any differentiable function f.

Example 9.2.2 Simplify the following expressions.

a. $\frac{d}{dx}(3x^4)$

b. $\frac{d}{dx}(-x)$

Example 9.2.3 Show that $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$ supposing that both f and g are differentiable.

9.3 Exponential and Trigonometric Functions

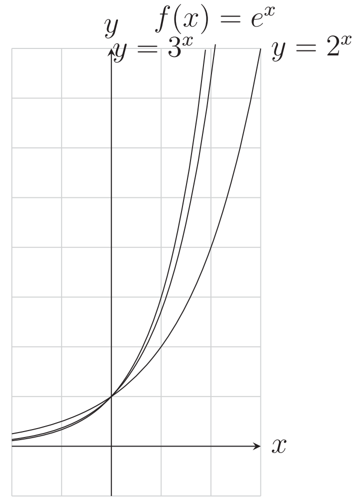
Definition 9.3.1

The number e is defined to be the number which satisfies the equation $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$.

Note:

$$\begin{aligned}
\lim_{h \to 0} \frac{e^h - 1}{h} &= \lim_{h \to 0} \frac{e^{0+h} - e^0}{h} \\
&= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \\
&= f'(0) \\
&= \text{ the slope of } e^x \text{ when } x = 0.
\end{aligned}$$

Example 9.3.1 Show that if $f(x) = e^x$ then $f'(x) = e^x$ using the definition of e.



Example 9.3.2 If $f(x) = e^x - x$, find f' and f''.

Example 9.3.3 Show that $\frac{d}{dx}\sin(x) = \cos(x)$.

Theorem 9.3.1

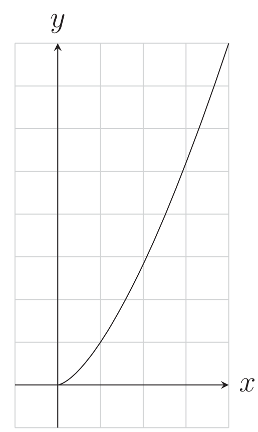
- $\frac{d}{dx}\cos(x) = -\sin(x)$ which may be derived using the same method that we used to show $\frac{d}{dx}\sin(x) = \cos(x)$.
- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$
- $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
- $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

9.4 Tangent and Normal Lines

Definition 9.4.1

The **normal line** to a curve C at a point P is the line through P that is perpendicular to the tangent line at P.

Example 9.4.1 Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$. Draw the tangent and normal lines on the same coordinate plane as $y = x\sqrt{x}$ shown below.



Example 9.4.2 Determine the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.