

10.1 The Product Rule

Theorem 10.1.1

The product rule for differentiation:

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)] = f(x)g'(x) + g(x)f'(x)$$

Proof

Example 10.1.1 Use the product rule to differentiate the following functions.

a. $f(x) = xe^x$

b. $f(t) = \sqrt{t}(a + bt)$

10.2 The Quotient Rule

Theorem 10.2.1

The quotient rule for differentiation:

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

Proof

Example 10.2.1 Use the quotient rule to find the derivative of the following function.

$$y = \frac{x^2 + x - 2}{x^3 + 6}$$

10.3 Product and Quotient Rules with Trigonometric Functions

Theorem 10.3.1

$$\frac{d}{dx}(\tan(x)) = \sec^2(x).$$

Proof

Example 10.3.1 Use the quotient rule AND product rule together to find the following derivative.

$$\frac{d}{dx} \left(\frac{x^3 e^x}{\cos(x)} \right)$$