11.1 The Chain Rule

Theorem 11.1.1

The chain rule for differentiation: If $h(x) = f(g(x))$ is a composite function, then $h'(x) = f'(g(x)) \cdot g'(x)$. In Leibniz notation: If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

Proof

Example 11.1.1 Find the derivative of the following functions using the chain rule.

a. $F(x) = \sqrt{x^2+1}$

b. $H(x) = 3\sin(2x^2)$

Theorem 11.1.2

$\frac{d}{dx}(a^x) = \ln(a)a^x$

Proof

Example 11.1.2 Take the derivative of the following functions by using the chain rule, product rule, and quotient rule together as appropriate.

a. $g(t) = \left(\dfrac{t-2}{2t+1}\right)^9$

b. $h(x) = (x^3 - 2)^2 \sin(e^{3x})$

11.2 Derivatives of Parametric Equations

Definition 11.2.1

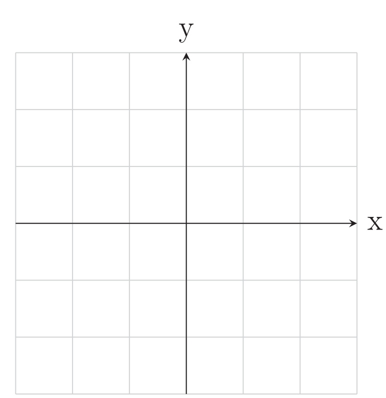
A **Parametric Curve** in two-dimensions is a graph whose points (x, y) are described by **Parametric Equations** in a third variable (usually t)

$$x = f(t) \quad \text{and} \quad y = g(t)$$

so that to find a point on the graph, you plug a t-value into both f and g to obtain the x and y-values of the point.

Example 11.2.1 Graph the parametric curve described by

$$x = 2\sin(2t) \quad \text{and} \quad y = 2\sin(t)$$



Note that since the x and y-values of the curve are given in terms of a third parameter t, to determine the slope at a point on the graph we cannot take the derivative in our normal fashion.

Theorem 11.2.1

Given parametric equations

$$x = f(t) \quad \textit{and} \quad y = g(t)$$

then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}.$$

Or, in prime-notation,

$$y'(x) = \frac{f'(t)}{g'(t)}.$$

Proof

Example 11.2.2 Find an equation of the tangent line to the parametric curve

$$x = 2\sin(2t) \qquad\qquad y = 2\sin(t)$$

at the point $(\sqrt{3}, 1)$. Where does this curve have horizontal tangents?