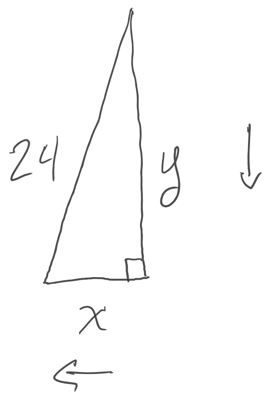
16.1 Related Rates with Known Equations

Example 16.1.1 A person is on top of a 24ft ladder that is leaning against a building. The bottom of the ladder begins to slip away from the building at a rate of 2ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?

Find $\left.\frac{dy}{dt}\right|_{x=8}$



$$\frac{dx}{dt} = 2\frac{ft}{sec}$$

$$x^2+y^2=24^2$$

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$4x + 2y\cdot\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-2x}{y}$$

$$x^2+y^2=24^2$$

$$8^2+y^2=24^2$$

$$y=\sqrt{24^2-8^2}$$

$$=\sqrt{512}$$

$$\left.\frac{dy}{dt}\right|_{x=8} = \frac{-2(8)}{y|_{x=8}}$$

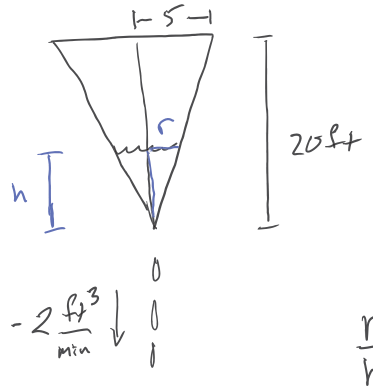
$$= \frac{-16}{\sqrt{512}} \approx -0.707$$

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$$

When the base of the ladder is 8ft from the wall the top of the ladder is sliding down at a rate of about 0.707 ft/s.

16.2 Determining Equations for Related Rates

Example 16.2.1 A tank filled with water is in the shape of an inverted cone 20ft high with a circular base (on top) whose radius is 5 feet. Water is running out of the bottom of the tank at the constant rate of 2 cubic feet per minute. How fast is the water level falling when the water is 8 feet deep?



Find $\left.\frac{dh}{dt}\right|_{h=8\text{ ft}}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{5}{20}$$

$$r = \frac{1}{4}h$$

$$V(h) = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 \cdot h$$

$$V = \frac{\pi}{48}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{16}h^2 \cdot \frac{dh}{dt}$$

$$-2 = \frac{\pi}{16}h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-32}{\pi h^2}$$

$$\left.\frac{dh}{dt}\right|_{h=8} = \frac{-32}{\pi \cdot 64} \approx -0.15915$$

When the height of the water is 8 ft, the height of the water is decreasing at a rate of about 0.15915 ft/min.

16.3 Strategy for Related Rates Problems

Steps to Success:

a. Draw a picture. It is **vital** that you recognize moving vs. non-moving parts!
   i. Label any **fixed** values with their value and unit.

   ii. Label any **changing** values with a **variable**. Note these variables are **functions of time**!

b. State any constant rates that are given in the problem using Leibniz notation.

c. State the rate which you are looking for using Leibniz notation and the "such that" symbol.

d. Determine and write an equation which relates the variables in question. You may include **fixed values for *non-moving parts* only!** It is important that your equation include the variable shown in the rate you are looking for.

e. Implicitly differentiate the equation with **respect to time** using Leibniz notation. This your **related rates equation.**

f. Plug in any **rate constants** at this point.

g. Solve for the rate which you are looking for.

h. Using proper Leibniz notation including the *such-that* bar, find the rate which you are looking for **at the given condition**.
   i. If there are variables remaining, use the such-that bar to indicate you need to determine that variable at the indicated condition.

   ii. Go back to the equation which relates the variable *before* implicit differentiation (step (d)) and determine the values of the variables needed to complete the evaluation.

i. State a conclusion using a complete sentence and proper units.