MTH 256 Lesson 14 - Three-Dimensional Systems of Linear ODEs

1. Find a general solution to $\dfrac{d\mathbf{y}}{dt} = \begin{pmatrix} -13 & 40 & -48 \\ -8 & 23 & -24 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{y}$. Determine how the system decouples, sketch the two-dimensional phase plane and one-dimensional phase line for the decoupled system, and give a rough sketch of the phase portrait of the system.

2. Find a general solution to $\dfrac{d\mathbf{y}}{dt} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & -2 & 3 \\ -1 & 3 & -1 \end{pmatrix} \mathbf{y}$ by first showing that $\mathbf{v}_1 = \langle 1, 1, 1 \rangle$ is an eigenvector of the coefficient matrix and finding its eigenvector. Why is this a necessary hint? Classify the system and find the general solution.