MTH 256 Lesson 19 - Undamped Forcing and Resonance

1. Consider the differential equation $\frac{d^2y}{dt^2} + 2y = \cos(\omega t)$. Determine the general solution to this equation. Compare graphs of the solution for $\omega = 0.5$, $\omega = 1$ and $\omega = 1.2$ for the solution when $y(0) = y'(0) = 0$. Factor the solution into a product of sine functions to determine the envelope curve, determine the frequence of the beats, and the frequency of the rapid oscillations. Note how the amplitude of the low frequency is dependent on the forcing frequency.

2. Determine the general solution to the equation $\dfrac{d^2y}{dt^2} + 2y = \cos(\sqrt{2}t)$. Graph the solution for when $y(0) = y'(0) = 0$.

3. Compute the solution of the initial-value problem $\dfrac{d^2y}{dt^2} + 4y = 3\sin(3t)$, $y(0) = 2$, $y'(0) = 0$. What is the frequency of the beats and rapid oscillations?