

MTH 256 Lesson 23 - Laplace Transforms on Discontinuous Functions

The Heaviside Function, is defined to be:

$$u_a(t) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}$$

1. Determine the Laplace Transformation for the Heaviside Function.
2. Determine the Laplace Transformation on the shift-function $g(t) = u_a(t)f(t - a)$, which takes the function f , shifts it by a , and turns it on at time $t = a$.
3. Use Laplace Transformations to find the solution to the initial value problem $y' = -y + 2u_3(t)$, $y(0) = 4$.

4. Use Laplace Transformations to find the solution to the initial value problem $y' = 2y + u_3 e^{-(t-3)}$, $y(0) = 0$.

5. Suppose $f(t)$ is a periodic function with period T ; that is, $f(t + T) = f(t)$ for all t . Show that $\mathcal{L}[f] = \frac{1}{1 - e^{-Ts}} \int_0^T f(t) e^{-st} dt$.
6. Compute the Laplace transform of the sawtooth function $z(t) = t - \lfloor t \rfloor$.