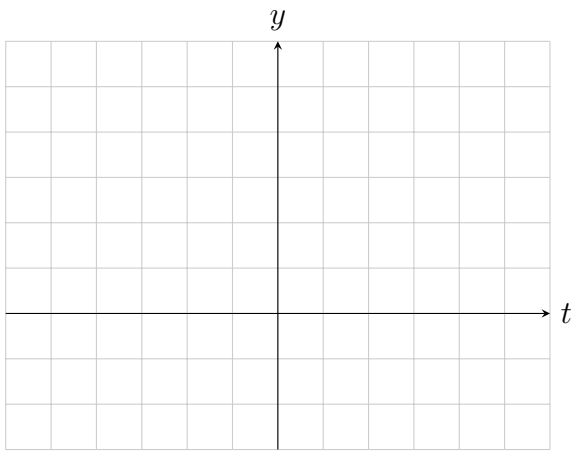
Math 253 Lesson 3 - The Logistic Equation

1. Solve the differential equation $\dfrac{dy}{dt} = k(y - b)$.

2. An engine is running at $100°C$ in a $21°C$ room when it is turned off. Five minutes later the temperture of the engine is found to be $70°C$. Find and solve the differential equation modeling this scenario. When will the engine have cooled to $40°C$?

3. An 80-kg skydiver jumps out of an airplane. Determine the terminal velocity if $k = 10$ kg/s for free fall.

4. Given the differential equation $\dfrac{dy}{dt} = 2y(3-y)$, determine the equilibrium solutions and plot a vector field for it. Plot a few specific solutions, both above, between, and below, the equilibrium solutions. Then find the particular solution satisfying $y(0) = 10$.



5. The population $P(t)$ of mosquito larvae growing in a tree hole increases according to the logistic equation with growth constant $k = 0.3\ day^{-1}$ and carrying capacity $A = 500$. Find a formula for the larvae population $P(t)$ assuming $P_0 = 50$ larvae. When will the population reach 200?

6. A rumor spreads through a small town. Let $y(t)$ be the fraction of the population that has heard the rumor at time t and assume that the rate at which the rumor spreads is proportional to the product of the fraction, y, of the population that has heard the rumor and the fraction $1 - y$ that has not yet heard the rumor. Find and solve the differential equation satisfied by y assuming that 10% of the population knows the rumor at $t = 0$ and 40% knows it 2 days later. When will 75% of the population know the rumor?

7. A tissue culture grows until it has a maximum area of m square centimeters. The area $A(t)$ of the culture at time t may be modeled by $\dfrac{dA}{dt} = k\sqrt{A}\left(1 - \dfrac{A}{M}\right)$ where k is a growth constant. Set $A = u^2$ and use substitution to simplify the differential equation. Then solve it using separation of variables.