

Math 253 Lesson 4 - First Order Linear Equations

1. Find the general solution of the following first-order linear equations.

a. $xy' - y = x^2 - x$

b. $y' + (\sec(x))y = \cos(x)$

2. Solve the following initial-value problems.

a. $xy' + y = e^x$ when $y(1) = 3$

b. $y' + \frac{x}{1+x^2}y = \frac{1}{(1+x^2)^{3/2}}$ when $y(1) = 0$

3. A 1000 Liter tank contains 500 L of water with a salt concentration of 10 g/L. Water with a salt concentration of 50 g/L flows into the tank at a rate of 60 L/min. The fluid mixes instantaneously and is pumped out at a rate of 40 L/min. Set up and solve the differential equation for $y(t)$, the amount of salt, in gallons, at time t , in minutes. What is the concentration of salt when the tank overflows?

4. Consider a series circuit consisting of a resistor of R ohms, an inductor of L henries, and a variable voltage source of $V(t)$ volts (time t in seconds). The current through the circuit, $I(t)$ (in amperes), satisfies the differential equation

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{1}{L}V(t)$$

Assuming that $R = 110$ ohms, $L = 10$ henries, and $V(t) = e^{-t}$ volts, solve the differential equation with $I(0) = 0$.