MTH 252 Lesson 4 - The Indefinite Integral and Antiderivatives Solutions

1. Evaluate the following indefinite integrals.

a. $\int 9x + 2\ dx$

$= \frac{9}{2}x^2 + 2x + C$

b. $\int z^{-4/5} - z^{2/3} + z^{5/4}\ dz$

$= 5z^{1/5} - \frac{3}{5}z^{5/3} + \frac{4}{9}z^{9/4} + C$

c. $\int \frac{12 - z}{\sqrt{z}}\ dz = \int 12z^{-1/2} - z^{1/2}\ dz$

$= 24z^{1/2} - \frac{2}{3}z^{3/2} + C$

d. $\int \frac{1}{3}\sin(x) - \frac{1}{4}\cos(x)\ dx$

$= -\frac{1}{3}\cos(x) - \frac{1}{4}\sin(x) + C$

e. $\int \theta + \sec^2(\theta)\ d\theta$

$= \frac{1}{2}\theta^2 + \tan(\theta) + C$

f. $\int \sec(x)\tan(x)\ dx$

$= \tan(x) + C$

g. $\int \frac{4}{x} - e^x\ dx$

$= 4\ln|x| - e^x + C$

h. $\int 8x - 4e^{5-2x}\ dx$

$= 4x^2 + 2e^{5-2x} + C$

2. Solve the initial value problem $\frac{dy}{dt} = t^{-3/2}$, $y(4) = -1$

$y(t) = -2t^{-1/2} + C$

$-1 = y(4) = -2(4)^{-1/2} + C$

$-1 = -1 + C \quad \Rightarrow \quad C = 0$

$y(t) = -2t^{-1/2}$

3. Solve the initial value problem $\frac{dy}{dt} = (4t+3)^{-2}$, $y(1) = 0$

$y(t) = \frac{-1}{4}(4t+3)^{-1} + C$

$0 = y(1) = -\frac{1}{28} + C$

4. Solve the initial value problem $f''(x) = x^3 - 2x$, $f'(1) = 0$, $f(1) = 2$

$f'(x) = \frac{1}{c}$

5. Suppose a ball is dropped and it falls for 2 seconds before hitting the ground. Determine how far it falls, assuming an acceleration of gravity of $-9.8$ m/s$^2$ and no wind resistance.