MTH 252 Lesson 6 - The Fundamental Theorem of Calculus II Solutions

1. Find $\frac{d}{dx}\int_2^x 12t^2 - 8t\, dt$ by first taking the integral and then the derivative. What seems to be happening?!

$$\frac{d}{dx}\int_2^x \underbrace{12t^2 - 8t}\, dt$$

$$= \frac{d}{dx}\left[4t^3 - 4t^2\right]_2^x$$

$$= \frac{d}{dx}\left[4x^3 - 4x^2 - (4\cdot 8 - 4\cdot 4)\right]$$

$$= \frac{d}{dx}\left(4x^3 - 4x^2 - 16\right)$$

$$= \underbrace{12x^2 - 8x}$$

These are the same with a variable switch!

The derivative of the integral is the original function!

2. Given $F(x) = \int_0^x \sqrt{t^2 + t}\, dt$, find $F(0)$, $F'(0)$, and $F'(3)$.

$$F(0) = \int_0^0 \sqrt{t^2+t}\, dt = 0$$

$$F'(x) = \sqrt{x^2+x}$$

$F'(0$

3. Find $\frac{d}{dx}\int_a^{f(x)} g(t)\, dt$ using the chain rule.

4. Calculate the following derivatives:

a. $\dfrac{d}{ds} \int\limits_{-2}^{s} \tan\left(\frac{1}{1+u^2}\right)\, du$

b. $\dfrac{d}{dx} \int\limits_{1}^{x^2} \cos^3(t) dt$

5. A factory produces bicycles at a rate of $95 - 3t^2 - t$ bicycles per week (t in weeks). How many bicycles were produced from the beginning of week 2 to the end of week 3?

6. The rate (in liters per minute) at which water drains from a tank is recorded at half-minute intervals. Compute the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first 3 minutes.

t (min)	0	0.5	1	1.5	2	2.5	3
r (liters/min)	50	48	46	44	42	40	38