Math 253 Lesson 7 - Geometric Series

1. Rewrite the series $\sqrt{2} - \frac{2}{3} + \frac{2^{3/2}}{9} - \frac{4}{27} + ...$ using sigma notation in the form $\sum_{i=0}^{\infty} ar^i$ and $\sum_{i=1}^{\infty} ar^{i-1}$. For either of those forms, what is a and what is r?

2. Prove that $\sum_{i=0}^{\infty} ar^i = a + ar + ar^2 + ar^3 + ...$ converges to $\frac{a}{1-r}$ if $|r| < 1$ and diverges if $|r| \geq 1$.

3. For problem (1), above, determine if the series is convergent or divergent. If it is convergent, determine its sum.

4. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

a. $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + ...$

c. $1.2 + \sum_{i=1}^{\infty} 3\left(\frac{2}{3}\right)^i$

b. $\sum_{i=0}^{\infty} \frac{1}{5^i}$

d. $7 + 2 - \frac{4}{5} + \frac{8}{25} - \frac{16}{125} + ...$

5. Express the number as a ratio of integers.

(a) $1.5\overline{36} = 1.536363636...$

(b) $3.2\overline{24} = 3.224242424...$