

MTH 256 Lesson 9 - Eigenspaces Introduction

1. Given the differential equation $y'(t) = \lambda y(t)$, What is the general solution for $y(t)$?
2. Given a matrix, \mathbf{A} , what is the definition of its eigenvectors and eigenvalues? Use $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ as an example.
3. Given the system of differential equations $\mathbf{y}'(t) = \mathbf{A}\mathbf{y}(t)$, where \mathbf{A} is a 2×2 matrix, what is the general solution for $\mathbf{y}(t)$?

4. Given the system of differential equations $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -4 & -2 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, compute the eigenvalues and eigenvectors, sketch the direction field and straight-line solutions, and state the general solution to system. For each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$ - and $y(t)$ -graphs. Find the particular solution with initial condition $(x(0), y(0)) = (1, -7)$.

