Math 253 LP 13 - 9.2 Vectors

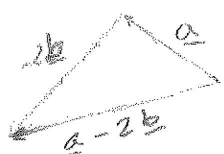
Solutions

Definition: A **vector** is directed line segment. That is, it is a mathematical object with both a magnitude and a direction often represented by an line in 2D or 3D with an arrow at one end to indicate its direction. A vector is notated using boldface, an arrow over the letter, or a line under the letter (British). I prefer the British notation.

Note that, generally speaking, a vector **does not have a fixed location** as its definition only indicates magnitude and direction without a fixed starting point. However, we often talk about vectors in a position by a **displacement** vector which has an **initial point** A and **terminal point** B which is indicated by $\mathbf{v} = \overrightarrow{AB}$. Note, there is often no indication of whether a vector is a displacement vector, but it is generally understood when that is the case.

Definition of Vector Addition: If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the **sum** $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

1. If $\mathbf{a}$ and $\mathbf{b}$ are the vectors shown in Figure 9, draw $\mathbf{a} - 2\mathbf{b}$



2. Find the vector represented by the directed line segment with initial point $A = (2, -3, 4)$ and terminal point $B = (-2, 1, 1)$.

$\overrightarrow{AB} = \langle -4, 4, -3 \rangle$

3. If $\mathbf{a} = \langle 4, 0, 4 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$, find $|\mathbf{a}|$ and the vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $3\mathbf{b}$, and $2\mathbf{a} + 5\mathbf{b}$

$|\underline{a}| = \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

$\underline{a} + \underline{b} = \langle 2, 1, 9 \rangle$

$\underline{a} - \underline{b} = \langle 4, -1, -1 \rangle$

$3\underline{b} = \langle -6, 3, 15 \rangle$

$2\underline{a} + 5\underline{b} = \langle -2, 5, 33 \rangle$

4. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 7\mathbf{k}$, express the vector $2\mathbf{a} + 3\mathbf{b}$ in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$2\underline{a} + 3\underline{b} = 14\vec{i} + 4\vec{j} + 15\vec{k}$

5. Find the **unit vector** in the direction of the vector $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$

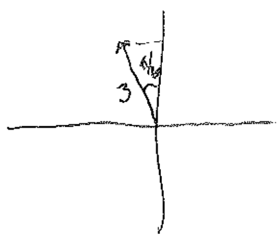
$|\underline{v}| = \sqrt{4+1+4} = 3$ $\qquad$ $\underline{u} = \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$

6. Find a vector that has the same direction as $\langle -2, 4, 2\rangle$ but has length 6.

$|\underline{v}| = \sqrt{4+16+4} = 2\sqrt{6}$ $\qquad$ $\underline{u} = \langle -\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6} \rangle$

$\underline{w} = 6\underline{u} = \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle$

7. If $\mathbf{v}$ lies in the 2nd quadrant and makes an angle of $\frac{\pi}{6}$ with the positive y-axis and $|\mathbf{v}| = 3$, find $\mathbf{v}$ in component form. What angle does $\mathbf{v}$ make with the positive x-axis?

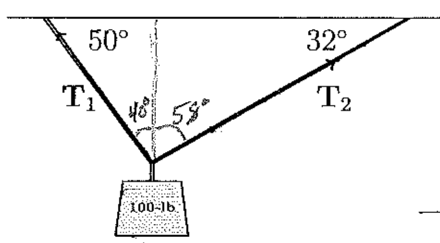


$\underline{v} = \langle 3\sin(\pi/6), 3\cos(\pi/6) \rangle$

$= \langle 3/2, 3\sqrt{3}/2 \rangle$

$\underline{v}$ makes an angle of $2\pi/3$ with the positive x-axis

8. A 100-lb weight hangs from two wires as shown in the figure below. Find the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and the magnitudes of the tensions.



Key

$\underline{T_1} = -a\cos(50°)\vec{i} + a\sin(50°)\vec{j}$

$\underline{T_2} = b\cos(32°)\vec{i} + b\sin(32°)\vec{j}$

$\underline{T_1} + \underline{T_2} - 100\vec{j} = 0$

$-a\cos(50°) + b\cos(32°) = 0$

$b = \frac{a\cos(50°)}{\cos(32°)}$

$a\sin(50°) + b\sin(32°) = 100$

$a\sin(50°) + \frac{a\cos(50°)\sin(32°)}{\cos(32°)} = 100$

$|T_1| = a = \dfrac{100}{\sin(50°) + \frac{\cos(50°)\sin(32°)}{\cos(32°)}} \approx 85.64$

$|T_2| = b = a\frac{\cos(50°)}{\cos(32°)} \approx 64.91$

$\underline{T_1} \approx -55.05\vec{i} + 65.60\vec{j}$ $\qquad$ $\underline{T_2} \approx 55.05\vec{i} + 34.40\vec{j}$