Math 111 LP 13 - Continuous Growth and Decay *Solutions*

1. Around 20 years ago some hikers in the Alps came across a frozen man (mummified) who has come to be known as Ötzi. Carbon dating was performed on the man and it was found that Ö had retained approximately 52.603% of the original amount of carbon−14 he would have had when he passed. Noting that carbon−14 has a half-life of about 5600 years, determine the century in which Ötzi died.

$A(t) = A_0 e^{kt}$

$A_0 = 1$ for 100% of original carbon

$.5 = 1 \cdot e^{k \cdot 5600}$

$\ln(.5) = k \cdot 5600$

$k = \frac{\ln(0.5)}{5600}$

$A(t) = e^{\ln(0.5)t/5600}$

$0.52603 = e^{\ln(0.5)t/5600}$

$\ln(0.52603) = \ln(0.5)t/5600$

$t = 5600\ln(0.52603)/\ln(0.5) \approx 5189.98$

Ötzi died about 5200 years ago, or sometime in the 33rd Century B.C.

2. Suppose you start off with a fungus which occupies 3 cubic centimeters of of a piece of rotting wood. After 5 days the fungus has spread to occupy 6 cubic centimeters of the rotting wood. Supposing the fungus is growing exponentially and uninhibited: a) Determine the continuous growth rate of the fungus per day. b) Write an explicit function which models the size of the fungus with respect to number of days you've been studying it. c) Find out how long it will take for the fungus to reach a size of 115 cubic centimeters.

$A(t) = 3e^{kt}$

$6 = 3e^{k \cdot 5}$

$2 = e^{k \cdot 5}$

$\ln(2) = k \cdot 5$

$k = \ln(2)/5$

≈ 0.1386

a) The continuous growth rate is about 13.86% per day

$A(t) = 3e^{\ln(2)t/5}$ ← (b)

$115 = 3e^{\ln(2)t/5}$

$\ln(115/3) = \ln(2)t/5$

$t = \frac{5\ln(115/3)}{\ln(2)} \approx 26.3$

c) It will take about 26.3 days for the fungus to reach 115 c.c.s.