Math 256 LP 14 - 7.1 Laplace Transforms and Inverse Transforms

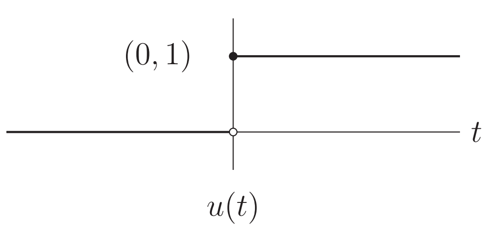
1. Apply the definition of a Laplace Transform directly to find $F(s)$ for the following functions.

a. $f(t) = c$

d. $f(t) = e^{at}$

b. $f(t) = t^n$

e. $f(t) = \sin(kt)$

c. 

2. Here's a handy list of pre-worked-out Laplace Transforms that you can just use as you please!

- $\mathcal{L}\{c\} = \frac{c}{s} \quad (s > 0)$

- $\mathcal{L}\{t\} = \frac{1}{s^2} \quad (s > 0)$

- $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad (n \geq 0) \quad (s > 0)$

- $\mathcal{L}\{t^a\} = \frac{\Gamma(a+1)}{s^{a+1}} \quad (s > 0)$ Where $\Gamma(a+1) = \int_0^\infty x^a e^{-x} dx$ is the gamma function which extends the concept of a factorial to all complex numbers except the non-positive integers. Want more? Do a web search.

- $\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad (s > a)$

- $\mathcal{L}\{\cos(kt)\} = \frac{s}{s^2+k^2} \quad (s > 0)$

- $\mathcal{L}\{\sin(kt)\} = \frac{k}{s^2+k^2} \quad (s > 0)$

- $\mathcal{L}\{\cosh(kt)\} = \frac{s}{s^2-k^2} \quad (s > |k|)$

- $\mathcal{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2} \quad (s > |k|)$

- $\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \quad (s > 0)$ Where $u(t-a)$ is the unit-step-function shifted via a.

3. Show that $\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$

4. Use the pre-determined transforms list to find the Laplace transforms of the following functions. A preliminary integration by parts may be necessary.

   a. $f(t) = \sqrt{t} + 3t$

   b. $f(t) = t^{3/2} - e^{-10t}$

5. Use the pre-determined transforms list to find the inverse Laplace transforms of the following functions.

   a. $F(s) = \frac{1}{s} - \frac{2}{s^{5/2}}$

   b. $F(s) = \frac{10s-3}{25-s^2}$