Math 253 LP 18 - 9.7 Cylindrical and Spherical Coordinates Solutions

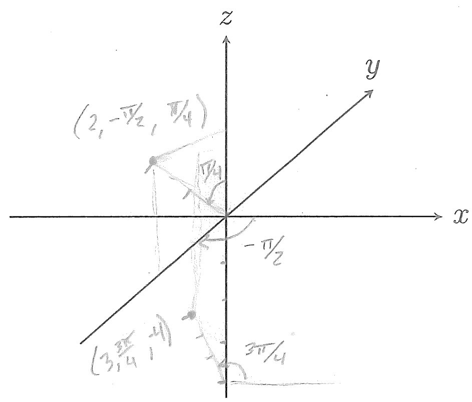
1. Plot the points whose coordinates are given. Then find the rectangular coordinates of the point.

$x = r\cos\theta$
$y = r\sin\theta$

a. $\overset{r}{(3}, \overset{\theta}{\tfrac{3\pi}{4}}, \overset{z}{-2})_C = \left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, -2\right)_R$

$x = 3\cos(3\pi/4) = 3 \cdot -\frac{\sqrt{2}}{2}$

$y = 3\sin(3\pi/4) = 3\sqrt{2}/2$



$x = \rho\cos(\theta)\sin(\varphi)$
$y = \rho\sin(\theta)\sin(\varphi)$
$z = \rho\cos(\varphi)$

b. $\overset{\rho}{(2}, \overset{\theta}{-\tfrac{\pi}{2}}, \overset{\varphi}{\tfrac{\pi}{4}})_S = (0, -\sqrt{2}, \sqrt{2})_R$

$x = 2\cos(-\pi/2)\sin(\pi/4) = 0$

$y = 2\sin(-\pi/2)\sin(\pi/4) = -\sqrt{2}$

$z = 2\cos(\pi/4) = \sqrt{2}$

2. Change the point $(1, -1, 4)$ from rectangular to cylindrical coordinates and spherical coordinates.

$r^2 = x^2 + y^2 = 2 \qquad r = \sqrt{2} \qquad \tan(\theta) = \frac{y}{x} = -1 \Rightarrow \theta = -\pi/4$

$(1, -1, 4)_R = (\sqrt{2}, -\pi/4, 4)_C$

$\rho^2 = x^2 + y^2 + z^2 = 18 \qquad \rho = 3\sqrt{2}$

$\rho\cos(\varphi) = z \Rightarrow 3\sqrt{2}\cos(\varphi) = 4 \Rightarrow \cos(\varphi) = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$

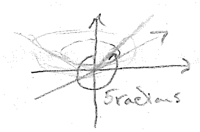
$\varphi = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$(1, -1, 4)_R = (3\sqrt{2}, -\pi/4, \cos^{-1}(2\sqrt{2}/3))$

3. Describe in words the surface whose equation is given by $\phi = 5$.

$\pi \approx 3.14$ $\quad \pi/2 \approx 1.57$ $\quad \frac{3\pi}{2} \approx 4.71$ $\quad$ so $\quad \frac{3\pi}{2} < 5 < 2\pi$

Cone in top-half of plane



4. Identify the surface whose equation is given.

a. $z = 4 - r^2$

$z = 4 - x^2 - y^2$

$z - 4 = -x^2 - y^2$

Downward facing Elliptic Paraboloid shifted up 4.

b. $\rho^2(\sin^2\phi \sin^2\theta + \cos^2\phi) = 9$

$\rho^2\left(\frac{y^2}{\rho^2} + \frac{z^2}{\rho^2}\right) = 9$

$y^2 + z^2 = 9$

cylinder with $r = 3$
center axis being the x-axis

5. Write the equation in (a) cylindrical coordinates and (b) spherical coordinates.

$x^2 - y^2 - z^2 = 1$

Cylindrical: $r^2\cos^2\theta - r^2\sin^2\theta - z^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = z^2 + 1$

$r^2 = \dfrac{z^2 + 1}{\cos(2\theta)}$

spherical: $\rho^2\cos^2\theta\sin^2\varphi - \rho^2\sin^2\theta\sin^2\varphi - \rho^2\cos^2\varphi = 1$

$\rho^2\left[(\cos^2\theta - \sin^2\theta)\sin^2\varphi - \cos^2\varphi\right] = 1$

$\rho^2 = \dfrac{1}{\cos(2\theta)\sin^2\varphi - \cos^2\varphi}$