Solutions

Math 253 LP ~~2~~ - 8.2 Series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \dots = \frac{a}{1-r}$$

1. Find at least 10 partial sums of the series. Graph both the sequence of terms and the sequence of partial sums on the same screen. Does it appear that the series is convergent or divergent? It it is convergent, find the sum.

a. $\sum_{n=1}^{\infty} \frac{12}{(-5)^n} = \sum_{n=1}^{\infty} 12\left(\frac{-1}{5}\right)^n = -\frac{12}{5} + \frac{12}{25} - \frac{12}{125} + \dots$

$= \frac{-12/5}{1-(-1/5)} = \frac{-12/5}{6/5} = -2$

$S_1 = \frac{-12}{5}$

$S_2 = -1.92$

$S_3 = -2.016$

$S_4 = -1.9968$

$S_5 = -2.0006$

$S_6 = -1.99872$

$S_7 = -2.0000256$

$S_8 = -1.99999488$

$S_9 = -2.000001024$

$S_{10} = -1.9999997952$

b. $\sum_{n=1}^{\infty} (1.1)^{n+1} = (1.1)^2 + (1.1)^3 + (1.1)^4 + \dots$

Is divergent since $r > 1$

$S_1 = (1.1)^2 = 1.21$

$S_2 = 2.541$

$S_3 = 4.0051$

$S_4 = 5.61561$

$S_5 = 7.387171$

$S_6 = 9.3358881$

$S_7 = 11.47947691$

$S_8 = 13.837424601$

$S_9 = 16.4311670611$

$S_{10} = 19.2842837672$

2. Determine whether the ~~geometric~~ series is convergent or divergent. If it is convergent, find its sum.

a. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ Divergent

$S_1 = 1$

$S_2 = 1 + \frac{1}{2}$

$S_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} > 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$= 1 + \frac{2}{2}$

$S_8 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$> 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= 1 + \frac{3}{2}$

$S_{2^n} > 1 + \frac{n}{2} \to \infty$

b. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{A}{n} + \frac{B}{n+1}$

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$1 = A(n+1) + Bn$

$1 = (A+B)n + A$

$A = 1 \quad A + B = 0$

$B = -1$

$= \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

$= \lim_{k\to\infty} \sum_{n=1}^{k} \frac{1}{n} - \frac{1}{n+1}$

$= \lim_{k\to\infty} \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} \dots + \frac{1}{k-1} - \frac{1}{k} + \frac{1}{k} - \frac{1}{k+1}\right)$

$= \lim_{k\to\infty} \left(1 - \frac{1}{k+1}\right) = 1$

3. Express the number as a ratio of integers.

$1.53\overline{42} = 1.5342424242...$

$$= 1.53 + \frac{42}{10{,}000} + \frac{42}{10^6} + \frac{42}{10^8} + \cdots$$

$$= 1 + \frac{53}{100} + \frac{42/10{,}000}{1 - 1/10^2}$$

$$= \frac{153}{100} + \frac{42/10{,}000}{99/100} = \frac{153}{100} + \frac{42}{9900} = \frac{5063}{3300}$$

4. Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} 4\left(\frac{2x}{5}\right)^n = 4 + 4 \cdot \frac{2x}{5} + 4 \cdot \left(\frac{2x}{5}\right)^2 + \cdots$$

$$= \frac{4}{1 - \frac{2x}{5}} \cdot \frac{5}{5}$$

$$= \frac{20}{5 - 2x} \qquad \text{When } \left|\frac{2x}{5}\right| < 1$$

$$-1 < \frac{2x}{5} < 1$$

$$-5 < 2x < 5$$

$$-\frac{5}{2} < x < \frac{5}{2}$$