Math 253 LP 3 - 8.2 Series Applications Name:______________________________

1. The Fibonacci sequence was defined in Section 8.1 by the equations

$$f_1 = 1, \qquad f_2 = 1, \qquad f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 3$$

Show that:

$$\frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}$$

$$\frac{1}{f_{n-1}\cdot f_n} - \frac{1}{f_n f_{n+1}}$$

$$= \frac{1}{f_n}\left(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}}\right)$$

$$= \frac{1}{f_n}\left(\frac{f_{n+1} - f_{n-1}}{f_{n-1}f_{n+1}}\right)$$

Lemma: $f_{n+1} = f_n + f_{n-1}$

$\Rightarrow f_n = f_{n+1} - f_{n-1} \longrightarrow$

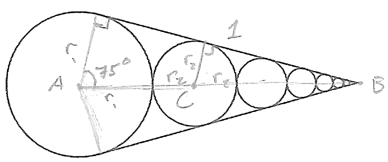
$$= \frac{f_n}{f_n}\cdot\frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_{n+1}}$$

Q.E.D.

2. In the figure there are infinitely many circles approaching the vertice of a 30° cone. Find the total area occupied by the circles if the lengths of the sides of the cone are 1.

$\tan 75° = \frac{1}{r_1}$

$\Rightarrow r_1 = 2-\sqrt{3}$



$$\sum A_{circles} = \sum_{i=1}^{\infty} \pi r_i^2$$

$$\cos(75°) = \frac{r_1}{|AB|} = \frac{r_2}{|CB|}$$

$$\Rightarrow |AB| = \frac{r_1}{\cos(75°)} \quad + \quad |CB| = \frac{r_2}{\cos(75°)}$$

Now $|AB| = r_1 + r_2 + |CB|$

$$\Rightarrow \frac{r_1}{\cos(75°)} - r_1 = r_2 + \frac{r_2}{\cos(75°)}$$

$$r_1\left(\frac{1}{\cos(75°)} - 1\right) = r_2\left(1 + \frac{1}{\cos(75°)}\right)$$

$$r_1(\sqrt{6}+\sqrt{2}-1) = r_2(1+\sqrt{6}+\sqrt{2})$$

$$\Rightarrow r_2 = \frac{\sqrt{6}+\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\, r_1$$

$$\Rightarrow r_i = \frac{\sqrt{6}+\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\, r_{i-1}$$

$$= \pi(2-\sqrt{3})^2 + \pi\left((2-\sqrt{3})\frac{\sqrt{6}+\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^2 + \cdots$$

$$+ \pi(2-\sqrt{3})^2\left(\frac{\sqrt{6}+\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^{2(i-1)} + \cdots$$

$$= \frac{\pi(2-\sqrt{3})^2}{1-\left(\frac{\sqrt{6}+\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^2}$$

$$\approx 0.345243225544$$