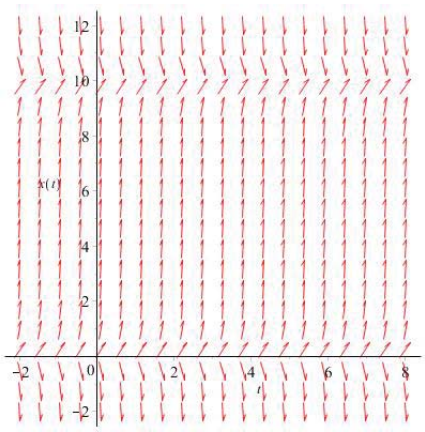
Math 256 LP 4 - 2.1 and 2.2 Population Models and Equilibrium Solutions

1. Solve the initial value problem $\frac{dx}{dt} = 10x - x^2$, $x(0) = 1$. Then sketch the graphs of several solutions of the given differential equation and highlight the indicated particular solution.



2. Suppose that when a certain lake is stocked with fish, the birth and death rates, β and δ are both inversely proportional to $\sqrt{P}$. Show that $P(t) = \left(\frac{1}{2}kt + \sqrt{P_0}\right)^2$ for some constant k. If $P_0 = 100$ and after 6 months there are 169 fish in the lake, how many will there be after 1 year?

3. Given $\frac{dx}{dt} = f(x)$, first solve the equation $f(x) = 0$ to find the critical points of the given autonomous differential equation. Then analyze the sign of $f(x)$ to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the differential equation. Next, solve the differential equation explicitly for $x(t)$ in terms of t. Finally, sketch typical solution curves for the given differential equation and verify visually the stability of each critical point.

$\frac{dx}{dt} = 9 - x^2$