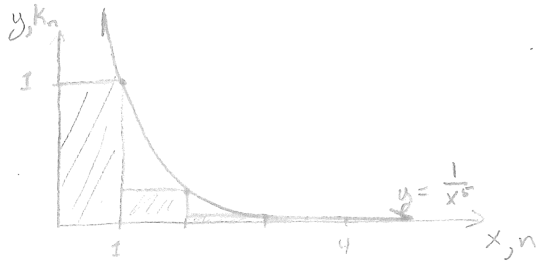
Math 253 LP 4 - 8.3 Integral and Comparison Tests   Name: Solutions

1. Use the integral test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^5}$$



$$\int_1^{\infty} x^{-5}\,dx = \lim_{t \to \infty} \int_1^{t} x^{-5}\,dx$$

$$= \lim_{t \to \infty} -\frac{1}{4} x^{-4} \Big|_1^t$$

$$= -\frac{1}{4} \lim_{t \to \infty} \left(\frac{1}{t^4} - 1 \right) = \frac{1}{4} < \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n^5} = 1 + \sum_{n=2}^{\infty} \frac{1}{n^5} < 1 + \int_1^{\infty} x^{-5}\,dx < \infty$$

So convergent.

2. Use the p-test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} 5n^{-1.1}$$

Convergent as $1.1 > 1$

$$= 5 \sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$$

3. Use the comparison test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{2^n - 1} < \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1/2}{1 - 1/2} = 1$$

These denominators are bigger than those so the terms on the left are smaller

Thus $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ is convergent

& in fact is less than 1.

4. Estimate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using the first 10 terms. How many terms are required to ensure that the sum is accurate to within 0.0005?

$$S_{10} = \sum_{n=1}^{10} \frac{1}{n^3} = 1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \frac{1}{216} + \frac{1}{343} + \frac{1}{512} + \frac{1}{729} + \frac{1}{1000}$$

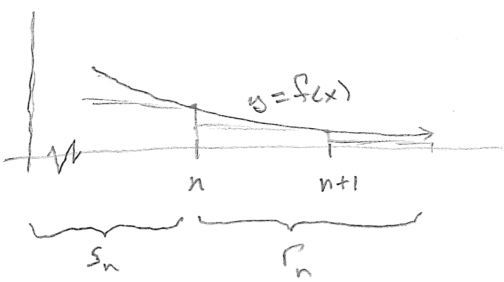
$$= \frac{19164113947}{16003008000} \approx 1.19753198567$$

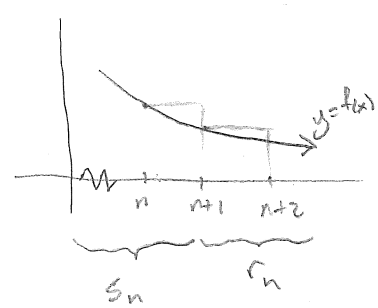
calc: $\Sigma(1/x\wedge 3, x, 1, 10)$

$$S = S_n + r_n$$

total sum; n^{th} partial sum; remainder of n^{th} partial sum. i.e.: n^{th} error

$$\int_{n+1}^{\infty} f(x)\,dx < r_n < \int_{n}^{\infty} f(x)\,dx$$





$$\int_{11}^{\infty} x^{-3}\,dx < r_{10} < \int_{10}^{\infty} x^{-3}\,dx$$

$$\lim_{t\to\infty} -\frac{1}{2}\left(\frac{1}{t^2} - \frac{1}{11^2}\right) < r_{10} < \lim_{s\to\infty} -\frac{1}{2}\left(\frac{1}{s^2} - \frac{1}{10^2}\right)$$

$$\frac{1}{242} < r_{10} < \frac{1}{200} = 0.005$$

$$S_{10} + \frac{1}{242} < S_{10} + r_{10} < S_{10} + \frac{1}{200}$$

$$1.20166421708 < S < 1.20253198567$$

$$S = 1.202098 \pm 0.000434$$

Want

$r_n < 0.0005$

Set:

$$\int_{n}^{\infty} x^{-3}\,dx < 0.0005$$

$$\lim_{t\to\infty} -\frac{1}{2}\left(\frac{1}{t^2} - \frac{1}{n^2}\right) < 0.0005$$

$$\frac{1}{2n^2} < \frac{5}{10{,}000}$$

$$10{,}000 < 10n^2$$

$$1000 < n^2$$

$$32 < n$$

Need $n > 32$ for this accuracy.