

Math 111 LP 4 - Function Algebra

1. Suppose $f(x) = 2x^2 + 1$ and $g(x) = -3x + 2$. Find the following:

$$\begin{aligned} \text{a. } (f+g)(2) &= f(2) + g(2) \\ &= 9 + (-4) \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{b. } (f \cdot g)(-3) &= f(-3) \cdot g(-3) \\ &= 19 \cdot 11 \\ &= 209 \end{aligned}$$

2. Suppose $f(x) = \frac{3}{(x+1)(x-2)}$ and $g(x) = \frac{1}{x^2-1}$. Find $\left(\frac{g}{f}\right)(x)$. What are the domains of f , g , and g/f ?

$$\left(\frac{g}{f}\right)(x) = \frac{\frac{1}{x^2-1}}{\frac{3}{(x+1)(x-2)}}$$

$$\begin{aligned} &= \frac{1}{x^2-1} \cdot \frac{(x+1)(x-2)}{3} \\ &= \frac{(x+1)(x-2)}{3(x+1)(x-1)} = \frac{x-2}{3x-3} \end{aligned}$$

From here!

$$D_f = \{x \mid x \neq -1, 2\}$$

$$D_g = \{x \mid x \neq -1, 1\}$$

$$D_{g/f} = \{x \mid x \neq -1, 1, 2\}$$

3. Suppose $f(x) = x^2 - 2x - 3$. Find $\frac{f(x+h) - f(x)}{h}$.

$$f(z) = (z)^2 - 2(z) - 3$$

$$f(a) = a^2 - 2a - 3$$

$$f(\Delta) = \Delta^2 - 2\Delta - 3$$

$$f(cac) = (cac)^2 - 2(cac) - 3$$

$$\begin{aligned} f(x+h) &= (x+h)^2 - 2(x+h) - 3 \\ &= x^2 + 2xh + h^2 - 2x - 2h - 3 \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - 2x - 2h - 3 - (x^2 - 2x - 3)}{h}$$

$$= \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{h(2x + h - 2)}{h}$$

$$= 2x + h - 2$$

4. Suppose that $f(x) = x + 3$ and $g(x) = x^2 - 3$. Find the following:

$$\begin{aligned} \text{a. } (f \circ g)(2) &= f(g(2)) \\ &= f(1) \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b. } (f \circ g)(x) &= f(g(x)) \\ &= f(x^2 - 3) \\ &= (x^2 - 3) + 3 \\ &= x^2 \end{aligned}$$

5. Suppose that $f(x) = \frac{3}{x+3}$ and $g(x) = \frac{1}{3x-2}$. Find $(f \circ g)(x)$ and state its domain.

$$(f \circ g)(x) = f(g(x))$$

$$\begin{aligned} &= \frac{3}{\frac{1}{3x-2} + 3} \cdot \frac{3x-2}{3x-2} \\ &= \frac{9x-6}{1+9x-6} \\ &= \frac{9x-6}{9x-5} \end{aligned}$$

$$D_{f \circ g} = \left\{ x \mid x \neq \frac{2}{3}, \frac{5}{9} \right\}$$

6. Find functions f and g such that $f \circ g = H$ if $H(x) = 2(3x-1)^2$. In fact, find multiple solutions to this exercise.

Order of ops

x
 $3x$
 $3x-1$
 $(3x-1)^2$
 $2(3x-1)^2$

$$\begin{aligned} g(x) = x & \quad \& \quad f(x) = 2(3x-1)^2 & \quad \leftarrow \text{Trivial} \\ \hline g(x) = 3x & \quad \& \quad f(x) = 2(x-1)^2 \\ \hline g(x) = 3x-1 & \quad \& \quad f(x) = 2x^2 \\ \hline g(x) = (3x-1)^2 & \quad \& \quad f(x) = 2x \\ \hline g(x) = 2(3x-1)^2 & \quad \& \quad f(x) = x & \quad \leftarrow \text{Trivial} \end{aligned}$$