Math 256 WA 14 - 6.2 and 6.3 Stability and Predator/Prey Models

Name: Solution

1. Given the system of differential equations $\frac{dx}{dt} = 30x - 3x^2 + xy$, $\frac{dy}{dt} = 60y - 3y^2 + 4xy$ which models the rates of changes of two interacting species populations, describe the type of x- and y-populations involved (exponential or logistic) and the nature of their interaction - competition, cooperation, or predation. Then find and characterize the system's critical points (type and stability). Determine what nonzero x- and y-populations can coexist. Then construct a phase plane portrait that enables you to describe the long-term behavior of the two populations.

Logistic, Cooperative.

$0 = x(30 - 3x + y) \quad 0 = y(60 - 3y + 4x)$

$x = 0 \quad y = 3x - 30 \quad\quad y = 0 \quad y = \frac{4}{3}x + 20$

$(0,0), (0,20), (10,0), (30,60)$

$$J(x,y) = \begin{pmatrix} 30 - 6x + y & x \\ 4y & 60 - 6y + 4x \end{pmatrix}$$

$J(0,0) = \begin{pmatrix} 30 & 0 \\ 0 & 60 \end{pmatrix} \quad \lambda_1 = 30 \quad \underline{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \lambda_2 = 60 \quad \underline{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$J(0,20) = \begin{pmatrix} 50 & 0 \\ 80 & -60 \end{pmatrix} \quad \lambda_3 = 50 \quad \underline{v_3} = \begin{pmatrix} 11 \\ 8 \end{pmatrix} \quad \lambda_4 = -60 \quad \underline{v_4} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

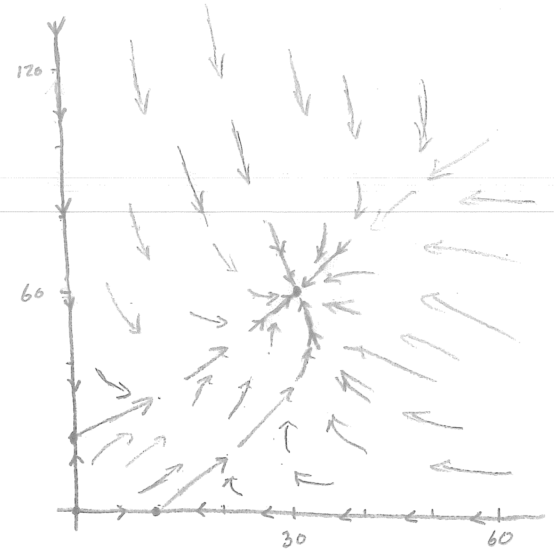
$J(10,0) = \begin{pmatrix} -30 & 10 \\ 0 & 100 \end{pmatrix} \quad \lambda_5 = -30 \quad \underline{v_5} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \lambda_6 = 100 \quad \underline{v_6} = \begin{pmatrix} 10 \\ 13 \end{pmatrix}$

$J(30,60) = \begin{pmatrix} -90 & 30 \\ 240 & -180 \end{pmatrix} \quad \begin{vmatrix} -90-\lambda & 30 \\ 240 & -180-\lambda \end{vmatrix} = \lambda^2 + 270\lambda + 9000$

$\lambda = -135 \pm 15\sqrt{41}$

$\lambda_7 = -135 + 15\sqrt{41} : \begin{pmatrix} 45 - 15\sqrt{41} & 30 \\ 240 & -45 - 15\sqrt{41} \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{30}{45 - 15\sqrt{41}} \\ 0 & 0 \end{pmatrix} \quad \underline{v_7} = \begin{pmatrix} -30 \\ 45 - 15\sqrt{41} \end{pmatrix}$

$\lambda_8 = -135 - 15\sqrt{41} : \quad \underline{v_8} = \begin{pmatrix} -30 \\ 45 + 15\sqrt{41} \end{pmatrix}$



The long term behavior is that if both populations start out non-zero they will tend to $(30,60)$.
If you start with only one of the species, it will tend to its logistic upper bound.