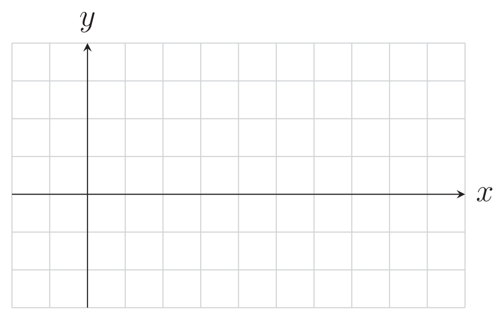
Math 111 WS 10, Properties of Logarithms

Name: ______________________________

1. Suppose $f(x) = \log_a(x)$ where $f(1) = 0$ and $f(9) = 2$. Sketch a graph of the situation and then determine the value of a.



2. Evaluate the following expressions.

a. $\log_a(1)$

b. $\log_a(a)$

c. $2^{\log_2(\pi)}$

d. $\log_{0.2}\left(0.2^{-\sqrt{2}}\right)$

e. $\ln e^{kt}$

f. $e^{\ln(1/2)\cdot t/3}$

3. Prove that $\log_a(MN) = \log_a(M) + \log_a(N)$

4. Prove that $\log_a(M^r) = r\log_a(M)$

5. Write the following expressions as a sum or difference of logarithms, pulling any exponents out of the logarithm by the end of the process.

a. $\log_a\left(x\sqrt{x^2+1}\right)$, $x > 0$

c. $\log_7\left(x^5 \cdot (x+3)^{2/3}\right)$, $x > 0$

b. $\ln\left(\dfrac{x^2}{(x-1)^3}\right)$, $x > 1$

d. $\log_a\left(\dfrac{\sqrt{x^2+1}}{x^3(x+1)^4}\right)$, $x > 0$

6. Write each of the following sums or differences of logarithms as a single logarithm.

a. $\log_a(7) + 4\log_a(3)$

b. $\frac{2}{3}\ln(8) - \ln(5^2 - 1)$

c. $\log_a(x) + \log_a(9) + \log_a(x^2 + 1) - \log_a(5)$

d. $21\log_3(\sqrt[3]{x}) + \log_3(9x^2) - \log_3(9)$

7. Approximate $\log_2(7)$ by first rewriting it as an exponential equation and then taking the ln of both sides.

8. Prove the change of base formula $\log_a(M) = \dfrac{\log_b(M)}{\log_b(a)}$ where b is any number you choose so long as $a \neq 1$, $b \neq 1$ and a, b, and M are all positive real numbers.

9. Approximate the following values by first applying the change of base formula.

a. $\log_5(89)$

b. $\log_{\sqrt{2}}(\sqrt{5})$