

Math 111 WS 11, Logarithmic and Exponential Equations

Name: Sdotians

1. Solve the following equations involving exponentials or logarithms.

a. $2 \log_5(x) = \log_5(9)$

$$\log_5(x^2) = \log_5(9)$$

$$x^2 = 9$$

$$x = \pm 3$$

-3 doesn't work!

The solution set is $\{3\}$

c. $\ln(x) = \ln(x+6) - \ln(x-4)$

$$\ln(x) = \ln\left(\frac{x+6}{x-4}\right)$$

$$x = \frac{x+6}{x-4}$$

$$x^2 - 4x = x + 6$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6, -1$$

The solution set is $\{6\}$.

b. $\log_5(x+6) + \log_5(x+2) = 1$

$$\log_5(x^2 + 8x + 12) = 1$$

$$x^2 + 8x + 12 = 5$$

$$x^2 + 8x + 7 = 0$$

$$(x+7)(x+1) = 0$$

$$x = -7, -1$$

The solution set is $\{-1\}$.

d. $\log(x) + \log(x-21) = 2$

$$\log(x^2 - 21x) = 2$$

$$x^2 - 21x = 10^2$$

$$x^2 - 21x - 100 = 0$$

$$(x-25)(x+4) = 0$$

$$x = 25, -4$$

The solution set is $\{25\}$.

e. $\log_4(x^2 - 9) - \log_4(x + 3) = 3$

$$\log_4 \left(\frac{(x-3)(x+3)}{x+3} \right) = 3$$

$$x-3 = 4^3$$

$$x = 67$$

The solution set
is $\{67\}$.

h. $8 \cdot 3^x = 5$

$$3^x = 5/8$$

$$\log_3(5/8) = x$$

The solution set is

$$\left\{ \log_3(5/8) \right\}$$

f. $\log_{1/2}(x-1) + \log_{1/2}(x+1) = -2$

$$\log_{1/2}(x^2 - 1) = -2$$

$$x^2 - 1 = \left(\frac{1}{2}\right)^{-2}$$

$$x^2 - 1 = 4$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

The solution set is $\{\pm\sqrt{5}\}$.

i. $5^{x-2} = 3^{3x+2}$

$$\ln(5^{x-2}) = \ln(3^{3x+2})$$

$$(x-2)\ln(5) = (3x+2)\ln(3)$$

$$\ln(5)x - 2\ln(5) = 3\ln(3)x + 2\ln(3)$$

$$\ln(5)x - 3\ln(3)x = 2\ln(3) + 2\ln(5)$$

$$x(\ln(5) - 3\ln(3)) = 2\ln(3) + 2\ln(5)$$

$$x = \frac{2\ln(3) + 2\ln(5)}{\ln(5) - 3\ln(3)} = \frac{\ln(225)}{\ln(5/27)}$$

The sol set is $\left\{ \frac{\ln(225)}{\ln(5/27)} \right\}$

g. $2^x = 5$

$$\ln(2^x) = \ln(5) \quad \text{or} \quad \log_2(5) = x$$

$$x \ln(2) = \ln(5)$$

$$x = \frac{\ln(5)}{\ln(2)}$$

The solution
set is

$$\left\{ \log_2(5) \right\}$$

The solution set is

$$\left\{ \frac{\ln(5)}{\ln(2)} \right\}$$

j. $\left(\frac{4}{3}\right)^{1-x} = 5^x$

$$(1-x)\ln(4/3) = x\ln(5)$$

$$\ln(4/3) - x\ln(4/3) = x\ln(5)$$

$$\ln(4/3) = x(\ln(4/3) + \ln(5))$$

$$x = \frac{\ln(4/3)}{\ln(20/3)}$$

The solution set

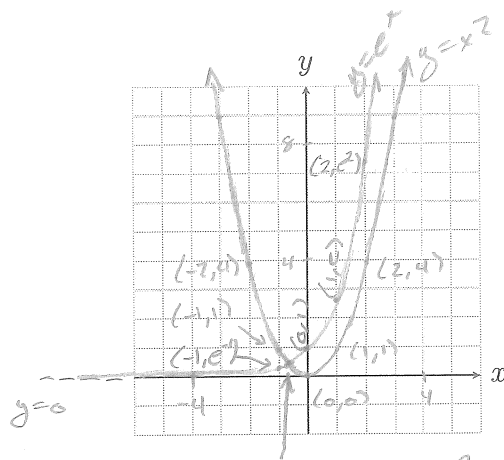
$$\text{is } \left\{ \frac{\ln(4/3)}{\ln(20/3)} \right\}$$

2. Solve the following equations by graphing. You may use your calculator on these problems.

a. $e^x = x^2$

Let $f(x) = e^x$
 & $g(x) = x^2$

The sol set
 is $\{x \mid x \approx -0.703\}$.



Intersection of about
 $(-0.703, 0.495)$

b. $e^x - \ln(x) = 4$

Let $f(x) = e^x - \ln(x)$
 & $g(x) = 4$

The sol set
 is $\{x \mid x \approx 0.05 \text{ or } x \approx 1.48\}$.

