Math 111 WS 13, Polynomial Functions   Name: Solutions

1. Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

a. $f(x) = 2 - 3x^4$

deg = 4

b. $g(x) = \sqrt{x}$

No, exponent not whole #

c. $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$

No, var. in den

d. $F(x) = 0$

deg. und.

e. $G(x) = 8$

Deg = 0

f. $H(x) = -2x^3(x-1)^2$

deg = 5

2. What is the standard form of a general polynomial?

$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ n^{th} degree

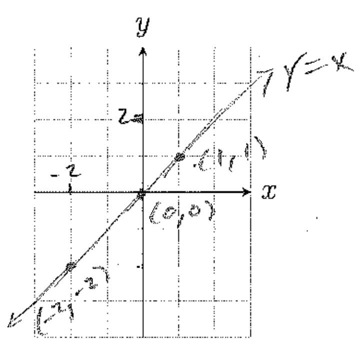
3. What is a **power function**?

A function of the form $pow_n(x) = x^n$

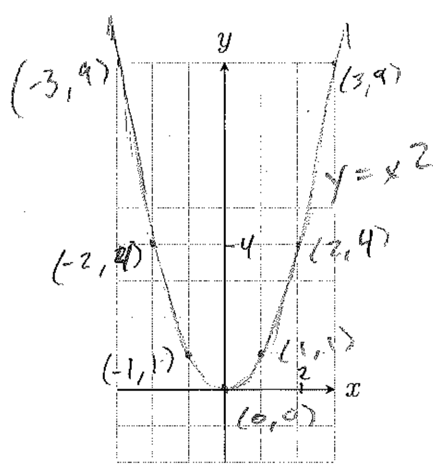
single term
coef. = 1
n^{th} degree.
Examples

4. Graph the first 6 power functions on the following coordinate planes.

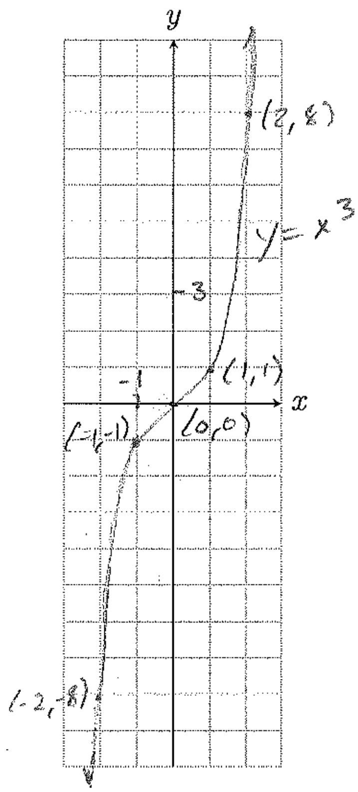
a. $linear(x) = x$



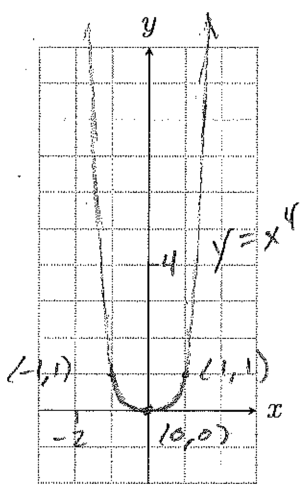
b. $quadratic(x) = x^2$



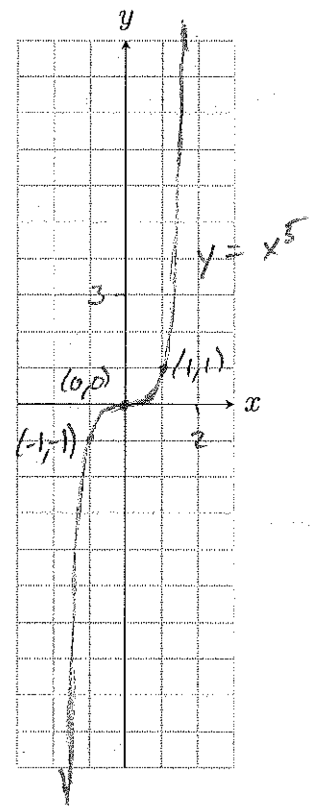
c. $cubic(x) = x^3$



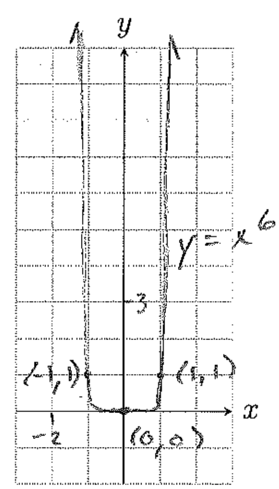
d. $quartic(x) = x^4$



e. $quintic(x) = x^5$



f. $sextic(x) = x^6$



5. What patters do we see arise that may help us memorize these?

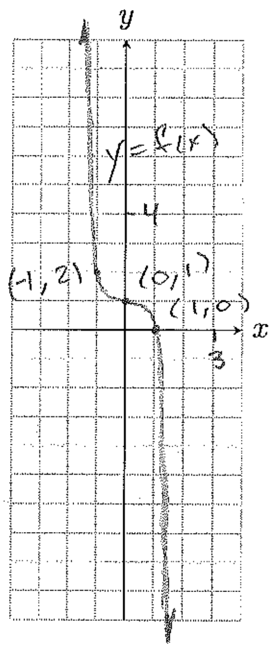
Odd powers   even

higher powers mean sharper corners (for power functions only?)

odd powers
have $(-1,-1), (0,0), (1,1)$

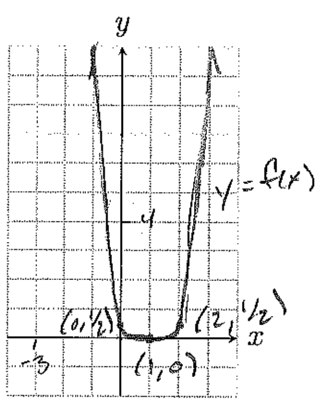
even powers
have $(-1,1), (0,0), (1,1)$

6. Graph $f(x) = 1 - x^5$ using transformations.



x^5	negate ys	add 1 to ys
$(-1,-1)$	$(-1,1)$	$(-1,2)$
$(0,0)$	$(0,0)$	$(0,1)$
$(1,1)$	$(1,-1)$	$(1,0)$

7. Graph $f(x) = \dfrac{1}{2}(x-1)^4$ using transformations.



x^4	right 1	mult. ys by 1/2
$(-1,1)$	$(0,1)$	$(0,1/2)$
$(0,0)$	$(1,0)$	$(1,0)$
$(1,1)$	$(2,1)$	$(2,1/2)$

8. Given the graph shown below in Figure 1, answer the following questions.

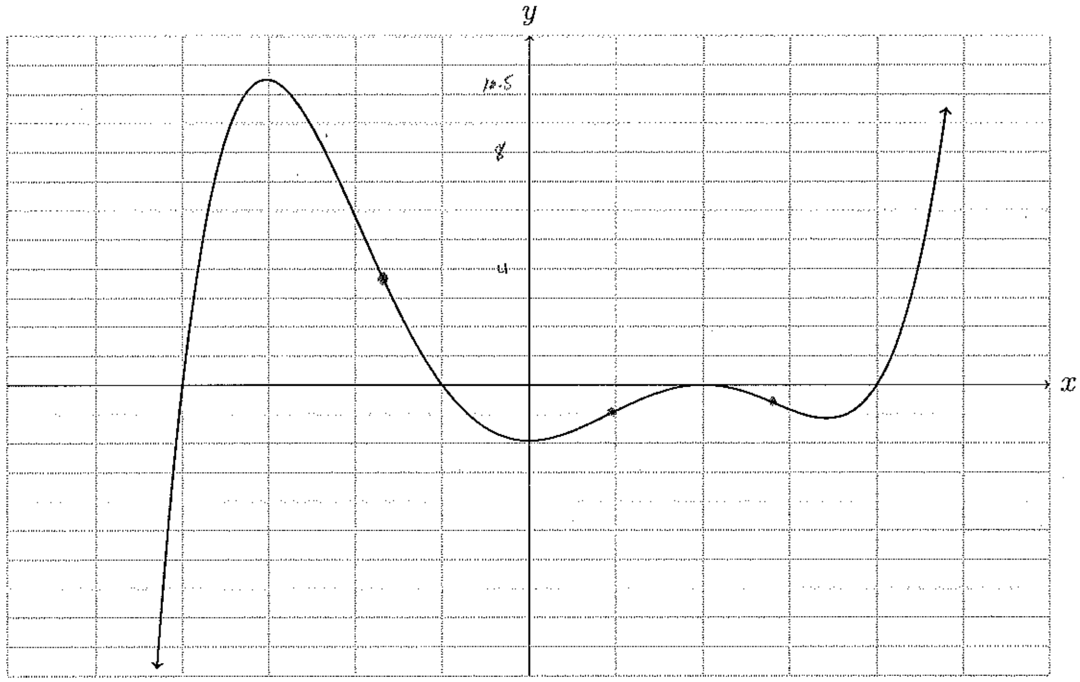


Figure 1: $y = f(x)$

a. What are the zeros of f and what is the multiplicity of each?

-4 : 1   -1 : 1   (first # is zero, 2nd is multiplicity)

2 : 2   4 : 1

b. How many turning points are there? What degree polynomial is this most likely?

4 turning points
probably 5th degree

c. Where is f positive and where is it negative?

pos: $(-4,-1) \cup (4,\infty)$
neg: $(-\infty,-4) \cup (-1,2) \cup (2,4)$

d. Where is f increasing and where is it decreasing?

inc: $(-\infty,-3) \cup (0,2) \cup (3.3,\infty)$
dec: $(-3,0) \cup (2,3.3)$

e. What and where are the local mins and maxs?

mins: -2 at $x=0$, -1.1 at $x \approx 3.3$
maxs: 10.5 at $x=-3$, 0 at $x=2$

f. Is there an absolute max or min? If so what and where?

None.

g. Where is f concave up and where is it concave down?

up: $(-1.7,.9) \cup (2.8,\infty)$
down: $(-\infty,-1.7) \cup (.9,2.8)$

S1 Use long division to divide the following polynomials.

a. $\dfrac{12x^3 - 24x^2 + 3x + 16}{6x + 3}$

$$\begin{array}{r} 2x^2 - 5x + 3 \\ 6x+3 \overline{)\, 12x^3 - 24x^2 + 3x + 16} \\ -(12x^3 + 6x^2) \\ \hline -30x^2 + 3x + 16 \\ -(-30x^2 - 15x) \\ \hline 18x + 16 \\ -(18x + 9) \\ \hline 7 \end{array}$$

1

2

3

$$\frac{12x^3 - 24x^2 + 3x + 16}{6x+3} = 2x^2 - 5x + 3 + \frac{7}{6x+3}$$

b. $\dfrac{10x^3 - 2x^2 + 3x - 11}{5x - 6} = 2x^2 + 2x + 3 + \dfrac{7}{5x-6}$

$$\begin{array}{r} 2x^2 + 2x + 3 \\ 5x-6 \overline{)\, 10x^3 - 2x^2 + 3x - 11} \\ -(10x^3 - 12x^2) \\ \hline 10x^2 + 3x - 11 \\ -(10x^2 - 12x) \\ \hline 15x - 11 \\ -(15x - 18) \\ \hline 7 \end{array}$$

S2 Given the function $g(x) = x^3 - 8x^2 + 18x - 12$, express g as a product of linear factors given that 2 is a zero of g.

If 2 is a zero then $(x-2)$ must be a factor:

$$\begin{array}{r} x^2 - 6x + 6 \\ x-2 \overline{)\, x^3 - 8x^2 + 18x - 12} \\ -(x^3 - 2x^2) \\ \hline -6x^2 + 18x - 12 \\ -(-6x^2 + 12x) \\ \hline 6x - 12 \\ -(6x - 12) \\ \hline 0 \end{array}$$

$$\begin{aligned} g(x) &= x^3 - 8x^2 + 18x - 12 \\ &= (x-2)(x^2 - 6x + 6) \\ &= (x-2)(x - (3+\sqrt{3}))(x - (3 - \sqrt{3})) \\ &= (x-2)(x - 3 - \sqrt{3})(x - 3 + \sqrt{3}) \end{aligned}$$

$$0 = x^2 - 6x + 6$$

$$\begin{aligned} x &= \frac{6 \pm \sqrt{36 - 4(1)(6)}}{2} \\ &= \frac{6 \pm \sqrt{12}}{2} \\ &= \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3} \end{aligned}$$

9. Suppose you know a polynomial has degree 3 and has three zeros, namely -3, 2, and 5.

a. Find a polynomial which fits these criteria.

$f(x) = (x+3)(x-2)(x-5)$

b. Verify this using a graphing utility.

See Desmos

c. If $(0, 60)$ is the y intercept, what does this end up forcing f to be?

$f(x) = a(x+3)(x-2)(x-5)$

$60 = a(0+3)(0-2)(0-5)$

$60 = a(3)(-2)(-5)$

$60 = a(30)$

$a = 2$

$f(x) = 2(x+3)(x-2)(x-5)$

10. Given the polynomial

$$f(x) = 5(x-2)(x+3)^2\left(x - \frac{1}{2}\right)^4,$$

answer the following questions.

a. What is the degree of the polynomial? What basic function does it most resemble?

Deg: 7 most resembles $y = 5x^7$

b. How many turning points, at most, does f have?

A most 6.

c. List the zeros and the multiplicity of each.

zero: 2 mult: 1 zero: $\frac{1}{2}$ mult: 4

zero: -3 mult: 2

d. What are the x- and y-intercepts of f?

x-ints: $(2,0)$, $(-3,0)$, $(\frac{1}{2}, 0)$

y-ints: $(0, \quad)$

$f(0) = 5(-2)(3)^2(-\frac{1}{2})^4 = -\frac{45}{8}$

$3x^2 - 4x = x(3x - 4)$

11. Given the polynomial $f(x) = x^2(x-2)$, complete the following.

a. What is the degree of the polynomial? What basic function does it most resemble?

Deg 3, resembles $y = x^3$

b. How many turning points, at most, does f have?

At most 2.

c. What are the zeros of f and the multiplicity of each?

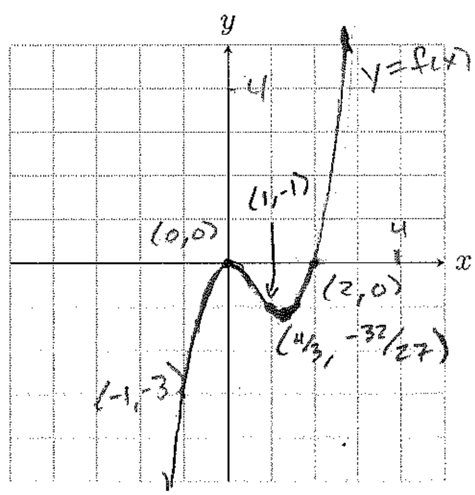
Zeros	multiplicity
0	2
2	1

d. What are the x- and y-intercepts of f?

x-ints: $(0,0)$ & $(2,0)$

y-ints: $(0,0)$

e. Graph f using the above information and by finding any additional points, if necessary, to draw a nice smooth curve.



$f(3) = 9$

$f(1) = -1$

$f(-1) = -3$

f. Use your calculator to determine the turning points of f.

There is a turning point at $(4/3, -32/27) \doteq (1.3, -1.2)$

12. Given the polynomial $f(x) = (2x+1)(x-3)^2$, complete the following.

a. What is the degree of the polynomial? What basic function does it most resemble?

deg 3, resembles $2x^3$

b. How many turning points, at most, does f have?

2 at most

c. What are the zeros of f and the multiplicity of each?

Zeros:	Mult.
$-\frac{1}{2}$	1
3	2

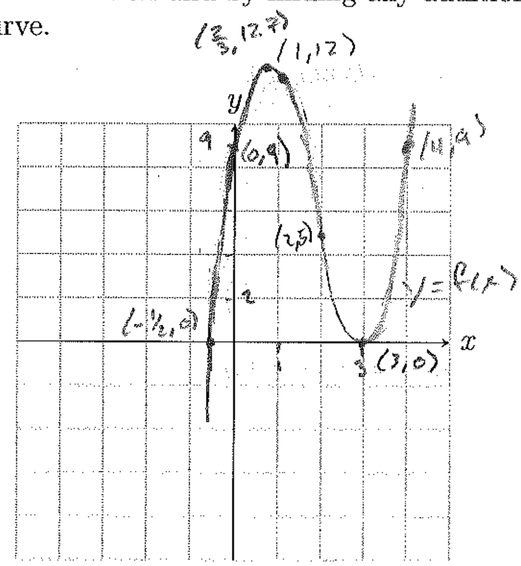
d. What are the x- and y-intercepts of f?

x-ints: $(-\frac{1}{2}, 0)$ & $(3, 0)$

y-int: $(0, 9)$

$f(0) = (1)(-3)^2 = 9$

e. Graph f using the above information and by finding any additional points, if necessary, to draw a nice smooth curve.



$f(1) = (3)(-2)^2 = 12$

$f(2) = (5)(-1)^2 = 5$

$f(4) = (9)(1)^2 = 9$

f. Use your calculator to determine the turning points of f.

$(\frac{2}{3}, 12.7)$ approximately

$(3, 0)$

13. Which of the graphs in Figure 2 could be the graph of

4th deg means 3 turning points tops.

neg y-int

$$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6?$$

positive coefficient so upward facing

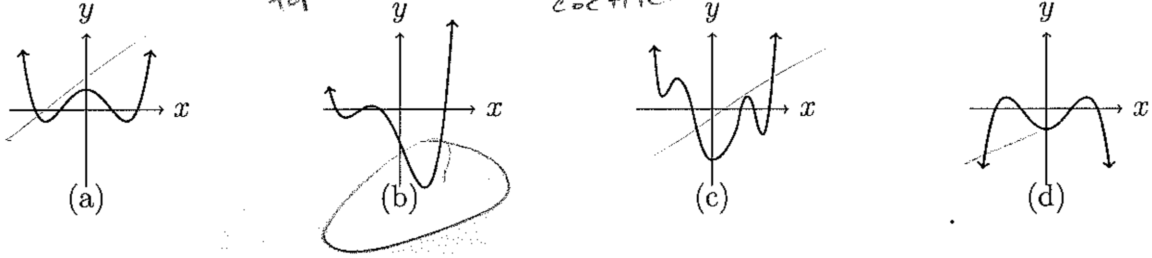


Figure 2

14. Use your calculator to graph $f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$ and then use it to answer the following questions.

a. What is the degree of the polynomial? What basic function does it most resemble?

Deg 3: $y = x^3$

b. How many turning points, at most, does f have?

2 at most

c. What are the zeros of f and the multiplicity of each?

Zero ≈ −3.79 mult: 1

d. Where is f positive and where is it negative?

pos: $\{x \mid x > -3.79\}$

neg: $(-\infty, -3.79)$

approximations

e. What are the x- and y-intercepts of f?

x-int ≈ (−3.79, 0)

y-int ≈ 2.4844

f. What are the turning points of f?

$(-2.28\bar{3}, 13.36)$ ← approx.

$(0.630, 1)$

g. What and where are any local maxs and mins?

local max ≈ 13.36 at $x = -2.28\bar{3}$

local min = 1, at $x \approx 0.630$

h. Where is f increasing and where is it decreasing?

inc: $(-\infty, -2.28\bar{3}) \cup (0.630, \infty)$

dec: $(-2.28\bar{3}, 0.630)$

Concave up: $(-.82\bar{6}, \infty)$

Concave down: $(-\infty, -.82\bar{6})$

inflection pnt: $(-.82\bar{6}, 7.18)$

15. Given the graph shown below in Figure 1, answer the following questions.

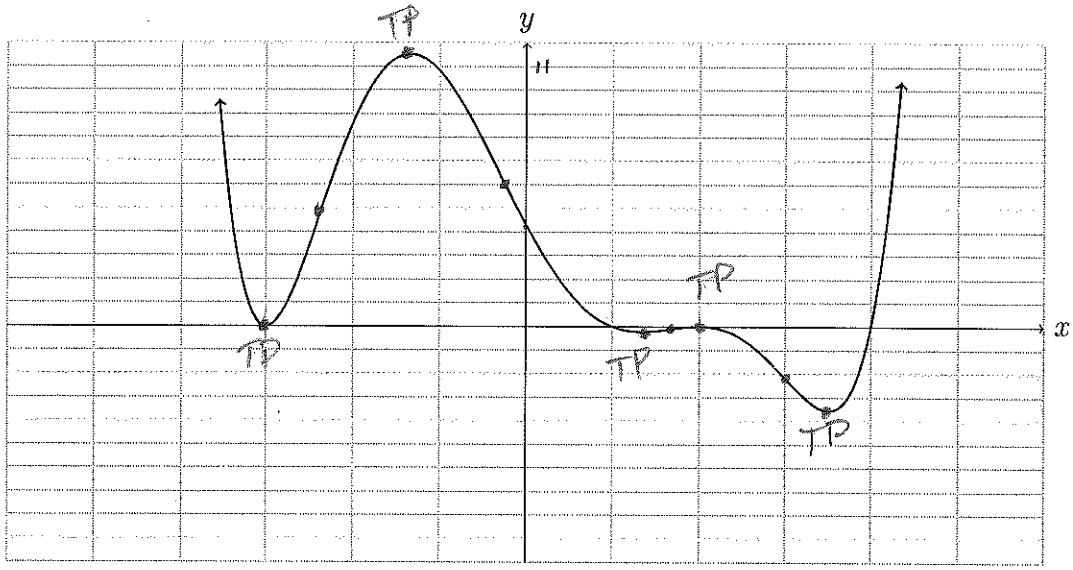


Figure 1: $y = f(x)$

a. What are the zeros of f and what is the multiplicity of each?

Zeros	-3	1	2	4
Mult.	2	1	2	1

b. How many turning points are there? What degree polynomial is this most likely?

T.P.s: 5

Deg: 6 most likely

c. Where is f positive and where is it negative?

Pos: $(-\infty,-3)\cup(-3,1)\cup(4,\infty)$

neg: $(1,2)\cup(2,4)$

d. Where is f increasing and where is it decreasing? approximations:

inc: $(-3,-1.3)\cup(1.3,2)\cup(3.5,\infty)$

dec: $(-\infty,-3)\cup(-1.3,1.3)\cup(2,3.5)$

e. What and where are the local mins and maxs?

mins: 0 @ $x=-3$, $-.3$ @ $x\approx 1.3$, -3.5 @ $x\approx 3.5$

maxs: 11.5 @ $x\approx -1.3$, 0 @ $x=2$

f. Is there an absolute max or min? If so what and where?

Abs max: none

Abs min ≈ -3.5 @ $x\approx 3.5$

g. Where is f concave up and where is it concave down?

Approximately.

Up: $(-\infty,-2.3)\cup(-.2,1.7)\cup(3,\infty)$

Down $(-2.3,-.2)\cup(1.7,3)$

h. What is the symbolic representation of the function f?

$f(x)=a(x+3)^2(x-1)(x-2)^2(x-4)$

$4.5=a(3)^2(-1)(-2)^2(-4)$

$a\approx 0.03125$

$f(x)=0.03125(x+3)^2(x-1)(x-2)^2(x-4)$