Math 111 WS 13, Polynomial Functions Name: _____

1. Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

   a. $f(x) = 2 - 3x^4$

   c. $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$

   e. $G(x) = 8$

   b. $g(x) = \sqrt{x}$

   d. $F(x) = 0$

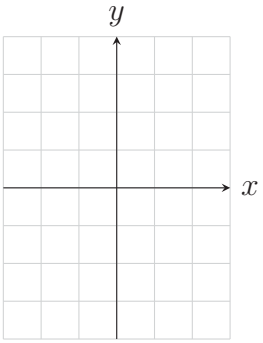
   f. $H(x) = -2x^3(x-1)^2$

2. What is the standard form of a general polynomial?

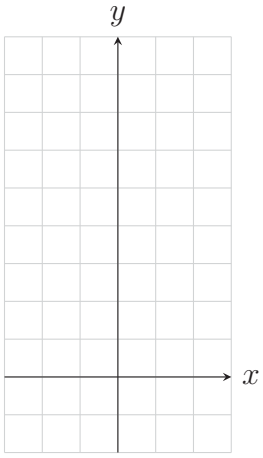
3. What is a **power function**?

4. Graph the first 6 power functions on the following coordinate planes.

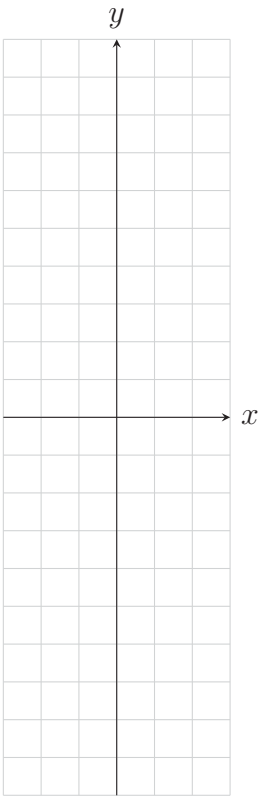
   a. $linear(x) = x$

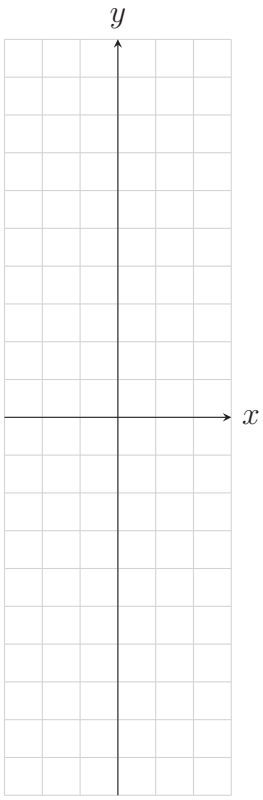
   b. $quadratic(x) = x^2$

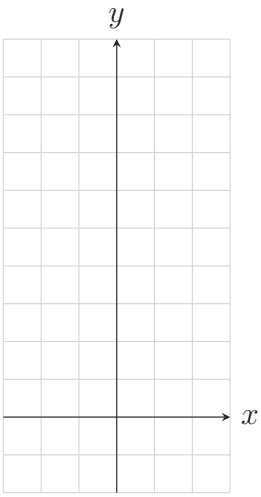
c. $cubic(x) = x^3$



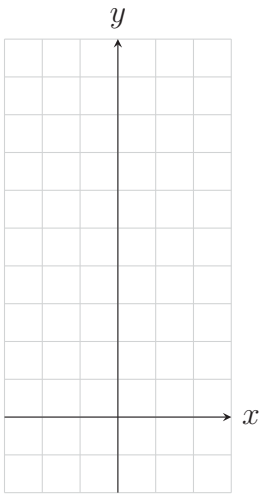

e. $quintic(x) = x^5$




d. $quartic(x) = x^4$

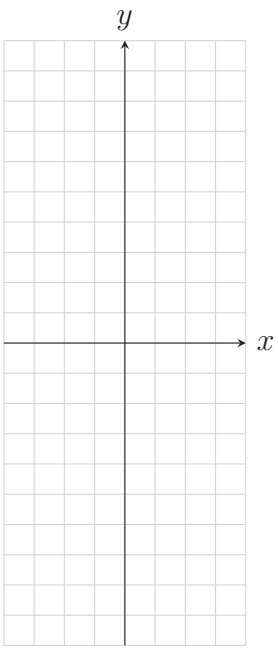



f. $sextic(x) = x^6$

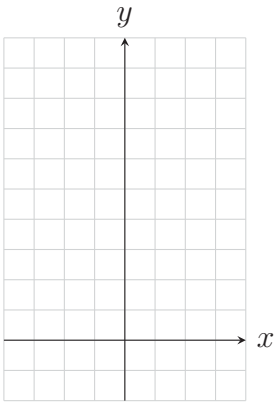


5. What patters do we see arise that may help us memorize these?

6. Graph $f(x) = 1 - x^5$ using transformations.




7. Graph $f(x) = \frac{1}{2}(x-1)^4$ using transformations.



8. Given the graph shown below in Figure 1, answer the following questions.

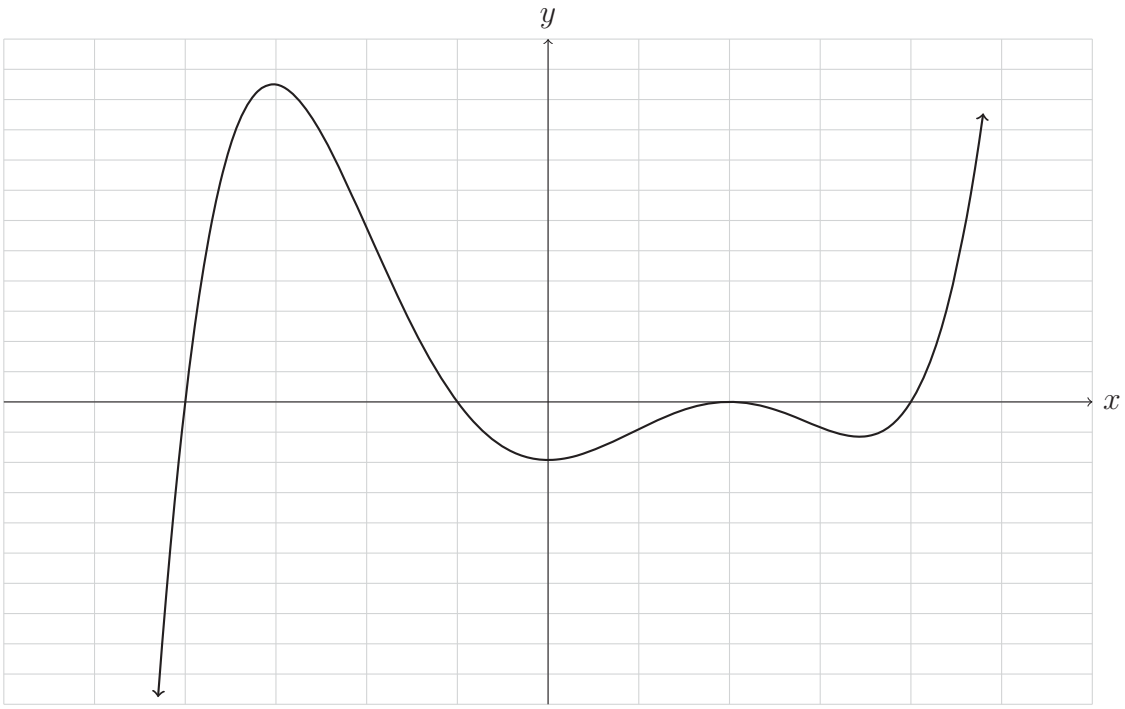



Figure 1: $y = f(x)$

a. What are the zeros of $f$ and what is the multiplicity of each?

b. How many turning points are there? What degree polynomial is this most likely?

c. Where is $f$ positive and where is it negative?

d. Where is $f$ increasing and where is it decreasing?

e. What and where are the local mins and maxs?

f. Is there an absolute max or min? If so what and where?

g. Where is $f$ concave up and where is it concave down?

S1 Use long division to divide the following polynomials.

a. $\dfrac{12x^3 - 24x^2 + 3x + 16}{6x + 3}$

b. $\dfrac{10x^3 - 2x^2 + 3x - 11}{5x - 6}$

S2 Given the function $g(x) = x^3 - 8x^2 + 18x - 12$, express $g$ as a product of linear factors given that 2 is a zero of $g$.

9. Suppose you know a polynomial has degree 3 and has three zeros, namely $-3$, 2, and 5.

   a. Find a polynomial which fits these criteria.

   b. Verify this using a graphing utility.

   c. If $(0, 60)$ is the $y$ intercept, what does this end up forcing $f$ to be?

10. Given the polynomial

$$f(x) = 5(x-2)(x+3)^2\left(x-\frac{1}{2}\right)^4,$$

answer the following questions.

   a. What is the degree of the polynomial? What basic function does it most resemble?

   b. How many turning points, at most, does $f$ have?

   c. List the zeros and the multiplicity of each.

   d. What are the $x$- and $y$-intercepts of $f$?

11. Given the polynomial $f(x) = x^2(x-2)$, complete the following.

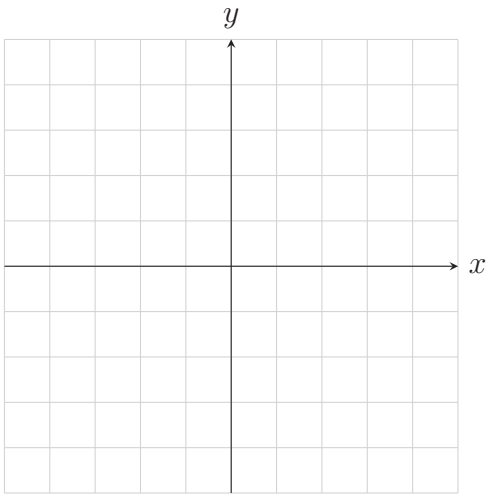
a. What is the degree of the polynomial? What basic function does it most resemble?

b. How many turning points, at most, does $f$ have?

c. What are the zeros of $f$ and the multiplicity of each?

d. What are the $x$- and $y$-intercepts of $f$?

e. Graph $f$ using the above information and by finding any additional points, if necessary, to draw a nice smooth curve.




f. Use your calculator to determine the turning points of $f$.

12. Given the polynomial $f(x) = (2x + 1)(x - 3)^2$, complete the following.

a. What is the degree of the polynomial? What basic function does it most resemble?

b. How many turning points, at most, does $f$ have?

c. What are the zeros of $f$ and the multiplicity of each?

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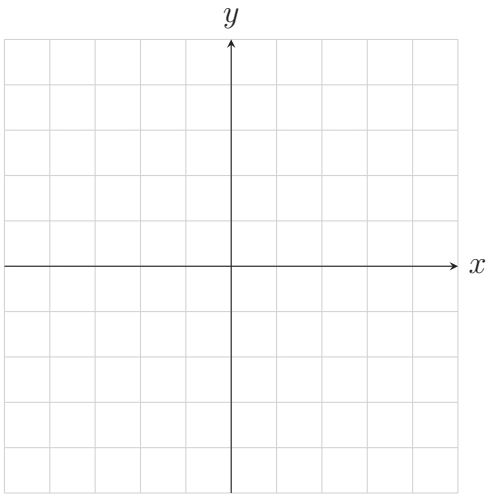



f. Use your calculator to determine the turning points of $f$.

13. Which of the graphs in Figure 2 could be the graph of

$$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6?$$

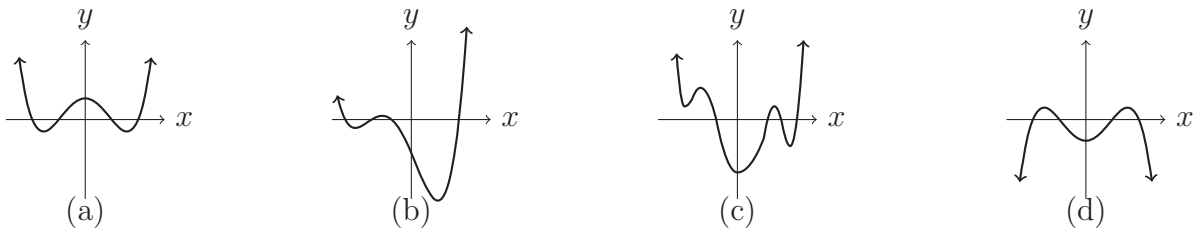



Figure 2

14. Use your calculator to graph $f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$ and then use it to answer the following questions.

    a. What is the degree of the polynomial? What basic function does it most resemble?

    b. How many turning points, at most, does $f$ have?

    c. What are the zeros of $f$ and the multiplicity of each?

    d. Where is $f$ positive and where is it negative?

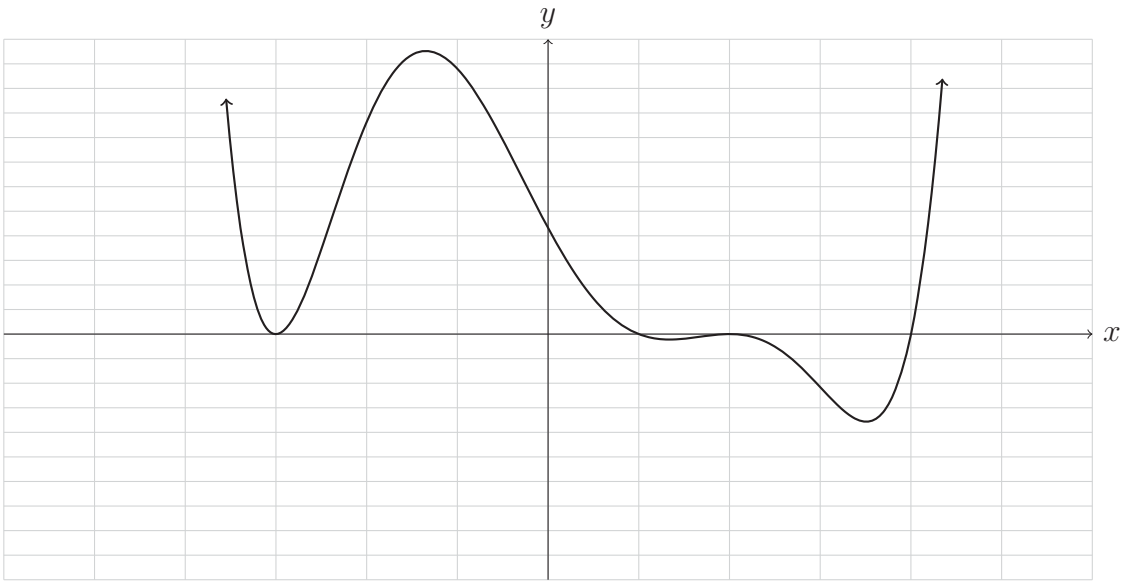
    e. What are the $x$- and $y$-intercepts of $f$?

    f. What are the turning points of $f$?

    g. What and where are any local maxs and mins?

    h. Where is $f$ increasing and where is it decreasing?

15. Given the graph shown below in Figure 1, answer the following questions.

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f. Is there an absolute max or min? If so what and where?

g. Where is $f$ concave up and where is it concave down?

h. What is the symbolic representation of the function $f$?