Math 251 In Class Worksheet 14

Related Rates: AKA "God awful story problems."

Steps to success:

a. Draw a picture. It is **vital** that you recognize moving vs. non-moving parts!

   i. Label any **fixed** values with their value and unit.

   ii. Label any **changing** values with a variable. Note these variables are **functions of time**!

b. State any constant rates that are given in the problem using Leibniz notation.

c. State the rate which you are looking for using Leibniz notation and the such that symbol.

d. Determine and write an equation which relates the variables in question. You may include **fixed values for *non-moving parts* only!** It is important that your equation include the variable shown in the rate you are looking for.

e. Implicitly differentiate the equation with **respect to time** using Leibniz notation. This is your **related rates equation.**

f. Plug in any **rate constants** at this point.

g. Solve for the rate which you are looking for.

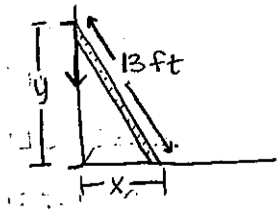
h. Use proper Leibniz notation including the *such-that* bar to find the rate which you are looking for **at the given condition**.

   i. If there are variables remaining, use the such-that bar to indicate you need to determine that variable at the indicated condition.

   ii. Go back to the equation which relates the variables *before* implicit differentiation (step d) and determine the values of the variables needed to complete the evaluation.

i. State a conclusion using a complete sentence and proper units.

1. A 13 foot ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/sec, how fast will the foot of the ladder be moving away from the wall when the top is 5 feet above the ground?



$$\frac{dy}{dt} = -2 \text{ ft/s}$$

find $\left.\dfrac{dx}{dt}\right|_{y=5}$

$$x^2 + y^2 = 13^2$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(13^2)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-2y\frac{dy}{dt}}{2x} = \frac{-y\, dy/dt}{x} \qquad \frac{dy}{dt} = -2$$

$$\frac{dx}{dt} = \frac{2y}{x} \qquad \left.\frac{dx}{dt}\right|_{y=5} = \frac{2\cdot 5}{x|_{y=5}} \qquad x^2 + 5^2 = 13^2$$

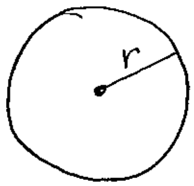
$$x = 12$$

$$\left.\frac{dx}{dt}\right|_{y=5} = \frac{10}{12} = 5/6 \text{ ft/s}$$

The foot of the ladder is moving away from the wall at 5/6 ft/s when y=5ft.

2. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 meters per second. How fast is the area of the spill increasing when the radius of the spill is 60 meters?

Picture



$\frac{dr}{dt} = 2 m/s$

State given information

find $\left.\frac{dA}{dt}\right|_{r=60m}$

state what you're asked to find

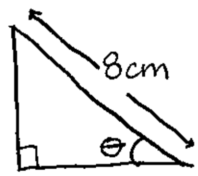
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ $\qquad \frac{dr}{dt} = 2 m/s$

$\left.\frac{dA}{dt}\right|_{r=60m} = 2\pi(60m) \cdot 2m/s = 754 m^2/s$

The area is increasing at a rate of $754 m^2/s$ when the radius is 60m.

3. The hypotenuse of a right triangle has a constant length of 8 cm. One of the acute angles of the triangle, θ, is growing at the rate of 1/2 radians each second. How is the area of the triangle changing when $\theta = \frac{\pi}{3}$?



* picture *

$h = 8\text{cm}$

$\frac{d\theta}{dt} = \frac{1}{2}$ radians/s

* constants *

Find $\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}}$

* find statement *

$A = \frac{1}{2}bh \qquad b = \cos(\theta)\cdot 8\text{cm} \qquad h = \sin(\theta)\cdot 8\text{cm}$

$$A = \frac{1}{2}(\cos(\theta)\,8\text{cm})(\sin(\theta)\,8\text{cm}) = 32(\sin(\theta))(\cos(\theta))$$

$$\frac{dA}{dt} = 32\left(\cos(\theta)\frac{d\theta}{dt}\cos(\theta) + (\sin(\theta))\cdot -\sin(\theta)\frac{d\theta}{dt}\right)$$

$$\frac{dA}{dt} = 32\left(\cos^2(\theta)\frac{d\theta}{dt} - \sin^2(\theta)\frac{d\theta}{dt}\right)$$

$$= 32\frac{d\theta}{dt}(\cos^2(\theta) - \sin^2(\theta)) \qquad \frac{d\theta}{dt} = \frac{1}{2}$$

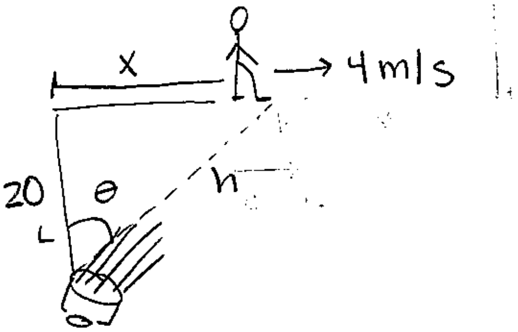
$$\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}} = 16\left(\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{\sqrt{3}}{2}\right)^2\right)$$

$$= 16\left(\frac{1}{4} - \frac{3}{4}\right)$$

$$= -8\ \text{cm}^2/\text{s}$$

The area is decreasing at a rate of $8\text{cm}^2/$ per second when $\theta = \frac{\pi}{3}$.

4. A man walks along a straight path at a speed of 4 feet per second. A searchlight is located on the ground 20 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 feet from the point on the path closest to the searchlight?



find $\left.\frac{d\theta}{dt}\right|_{x=15}$

$\frac{dx}{dt} = 4 \text{m/s}$

$L = 20\text{ft}$

$\tan\theta = \frac{x}{20\text{ft}}$ $\qquad$ $x = 15 \quad L = 20$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(x \cdot \frac{1}{20}\right)$$

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{20} \cdot \frac{dx}{dt} \qquad \frac{dx}{dt} = 4 \text{m/s}$$

$$\frac{d\theta}{dt} = \frac{1}{20\sec^2\theta} \cdot \frac{dx}{dt} = \frac{1}{20}\frac{dx}{dt}\cos^2\theta$$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$$\left.\frac{d\theta}{dt}\right|_{x=15} = \frac{1}{5}\cos^2\left(\theta\big|_{x=15}\right) = \frac{1}{5}\left(\frac{20}{\sqrt{15^2+20^2}}\right)^2$$

$$\left.\cos\theta\right|_{x=15} = \boxed{\frac{20}{\sqrt{15^2+20^2}}}$$

$$= \frac{80}{225+400} = \frac{16}{125} \text{ radians/s}$$

The searchlight is rotating at 16/125 radians per second when the man is 15ft from the point on the path closest to the searchlight.