Math 95 WS 1 Function Introduction Name: ______________________________

§2.1

What is a function? Well, it has a very formal and precise definition that I'll get to in a minute but I want you to think of a function as *a machine.* How does a machine work? Well, you put something into a machine, the machine does something to what you input and then it spits something out, the output! That is how we're going to be thinking of these functions, according to their **inputs** and **outputs**. Given some input, what is the machine going to output?

For example, a vending machine is a type of function. You input different combinations of a letter and a number and you get a different kind of candy out. For example, you press A2 and the machine will give you Fritos. If you press B4 then it gives you a Snickers Bar. Etc. However, if you press 3C, the machine won't give you anything! Why not?
So, we can list what the possible inputs are and then write a corresponding list of what the outputs will be. When we do this we use set notation for our list and us *ordered pairs* to write our inputs and corresponding outputs together! Our Candy machine might have a list like this:

$$\{(A1, Pretzels), (A2, Fritos), (A3, Fritos), (A4, Cheetos),$$
$$(B1, Butterfinger), (B2, ThreeMusketeers), (B3, Skittles), (B4, Snickers),$$
$$(C1, SpearmintGum), (C2CinnamonGum), (C3, Mints), (C4, LifeSavers)\}$$

So what do we call the list of things our machines accepts as inputs?

For the candy machine function described above, what is the domain? Use proper set notation.

And what do we call the list of things we get out of our machine (our outputs)?

For the candy machine function described above, what is the range? Use proper set notation.

Let's think about this: If a machine is given an input and then it spits out some output, if we put that same input into the machine again would we expect to get a different output?

For example, in the machine above, if I enter $C3$ should I always expect to get mints, or would it be acceptable to acquire Fritos?

So, that's one of the ideas of a function, that given any input it **only has one output!**

Now, this doesn't go the other way, given different inputs we could get the same output. For example, in our candy machine, both A2 and A3 gave us Fritos! Does this seem like an acceptable thing for our candy machine to do? Let's look at some lists of ordered pairs and determine which lists could be functions and which could not.

1. $\{(1,6),(2,6),(3,8),(4,9)\}$

2. $\{(6,1),(6,2),(8,3),(9,4)\}$

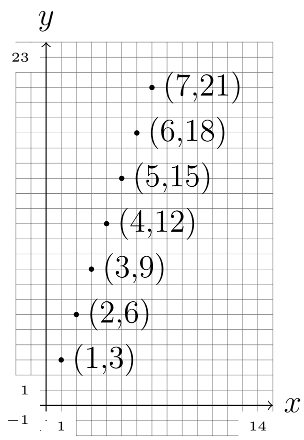
3. $\{(1,2),(3,4),(5,6),(5,8)\}$

4. $\{(1,2),(3,4),(6,5),(8,5)\}$

For the lists above which *are* functions, what is their domain and what is their range?

Note that we could have described the above lists of ordered pairs in a table or on a graph, like this:

x (yards)	y (feet)
1	3
2	6
3	9
4	12
5	15
6	18
7	21



We now need to address the symbolic representation of a function and **function notation**. Say I had the following linear equations:

$y = \frac{1}{2}x - 3$, $y = 2x + 5$ and $y = -x + 2$

Then I said, when $x = 1$, $y = 7$. Well, I could go and check each equation and decide which one I was referring to, but wouldn't it be easier to have a way of labeling each equation so that I could refer back to them quickly?! Of course!

So here's what we do, we'll call the first equation f, the second h and the third g. Then we'll use parenthesis next to these labels to tell us what the input for the equation is. That is, We'll write $f(x)$, $h(x)$ and $g(x)$. Note, this is NOT multiplication!

In fact, we'll write each equation in the following manner:

$f(x) = \frac{1}{2}x - 3$

$g(x) = 2x + 5$

$h(x) = -x + 2$

With the *implication* that the output of each is a y value. When we write it like this, we say that these **are functions!** We input an x value and get out a y value!

So, instead of saying, in one of these equations when $x = 1$, $y = 7$, I can say $g(1) = 7$. That is, when I put $x = 1$ into the function g, I get out $y = 7$! Much handier!

When we are dealing with a function who's input is x and it's output is y, what do we call x and y?

Let's look at this notation once more. Below this, label each part as I do on the board:

$$f(x) = 3x^2 - 6x - 9$$

Now, what does it mean to write $f(1)$? This means we take 1 and "input" it in for x, our independent variable, everywhere in the expression which makes up the function f. Then $f(1)$ will take on a y value. What y value does $f(1)$ equal in this case?

Note that $f(1)$ describes a y value, but when we write $f(1) = -12$ we see that we know a point on the graph of f. What is this point? Write it as an ordered pair.

5. Given the function f above, find the following and then write the corresponding ordered pair:

   a. $f(2)$

   b. $f(0)$

   c. $f(-1)$

So, we took the number in the parenthesis and used this as the input for x to get our output! Let's do one more:

6. Given $g(x) = \frac{x}{x+2}$, find each of the following and write the corresponding ordered pair:

   a. $g(2)$

   b. $g(-3)$

c. $g(0)$

d. $g(-1)$

Great! Now set up a table for each function that describes the points we found.

Next, let's look at some graphs and determine whether it constitutes a function or not. For the following problems, draw the graph that I put on the board and use the **vertical line test** to determine whether the graph represents a function.

Now, we spoke earlier about what the domain and range were for functions where we saw a list of their ordered pairs. but what if we have a function described symbolically, like $f(x) = \sqrt{x-3}$? Can we determine the domain and range here? Well, what do we know about square roots? We know that the *input* must be greater than or equal to zero, so we can set up the inequality $x - 3 \geq 0$ and use this to realize that x must be greater than or equal to 3. In set notation we would write $\{x | x \geq 3\}$.

What about the range? For now, we are going to use our calculators to determine the range of functions described symbollically. Go ahead and pull out your calculator and we will graph $f(x)$ and then use this to determine the range. What did we find? Use proper set notation.

7. Find the domain and range for the following functions. State using proper set notation.

   a. $f(x) = 5x$

   b. $g(x) = \frac{1}{x+2}$

   c. $h(x) = \sqrt{x+4}$

Let's look at a function modelling crutch length based on a persons height.

8. People who sustain leg injuries often require crutches. A proper crutch length can be estimated without using trail and error. The function L, given by $L(t) = 0.72t + 2$, outputs an appropriate crutch length in inches for a person t inches tall.

   a. Find $L(60)$ and interpret the result.

b. If one person is 70 inches tall and another person is 71 inches tall, what should be the difference in their crutch lengths?

To finish up section 2.1, let's look at different representations of a single function to see how they are connected.

9. Let function f square the input x and then subtract 1 to obtain the output y.

   a. Write a formula, or symbolic representation, for f.

   b. Make a table of values, or numerical representation, for f. Use $x = -2, -1, 0, 1, 2$.

   c. Sketch a graph, or graphical representation, of f.

   d. What is the domain and range of f?

§2.2 Let's do some line review.

For the next set of problems, write a function which describes a line with the given properties:

10. Passing through the point (3,2) with a slope of $m = 2$.

11. Passing through the points (-1,4) and (3,-2).

12. Having $y - int = (0, 3)$ and a slope of $m = -\frac{3}{7}$.

13. Of the above lines, which lines are increasing and which are decreasing? Write what it means for a line to be increasing in your own words.