Math 95 WS 2 Linear Functions Name: ______________________________

§2.2, 2.3, 2.4

In the past, we've looked at the equation of a line to be

$$y = mx + b.$$

Now we will be using function notation wherever appropriate and will thus write

$$f(x) = mx + b$$

instead. We call a function of this sort a **linear function.** For our purposes we will think of a linear function as one which has a *constant rate of change*. What this means is that the outputs increase at a constant rate whenever we increase the input by a certain amount. For example, in the following tables, note that as x increases by a fixed amount, so does y:

x	0	1	2	3
$f(x)$	10	15	20	25

x	-2	0	2	4
$f(x)$	4	1	-2	-5

x	1	3	5	9
$f(x)$	3	6	9	15

What is a little different about the third example above?

For all three of the above examples state a reduced rate of change.

If we look at some symbolic representations of functions, let's determine which are linear, which are not linear and why. When we give the why we want to base our logic upon the idea of a function described just above the tables.

$f(x) = 4 - 3x$ $\qquad$ $g(x) = 8$ $\qquad$ $h(x) = 2x^2 - 4$ $\qquad$ $j(x) = \frac{5}{x} + 1$

For the linear functions above, what is the rate of change for each?

Suppose it's 70° in your house and after turning off the heat the temperature starts to drop by 2° each hour. Determine a table which describes the temperture in your house, T, given how many hours have elapsed, t. Make the table go for 6 hours.

Can we model a symbolic linear function which describes this situation? Yes, of course we can! What would it be?

Suppose I decide to go for a leisurely walk and after traveling 2 miles realize that I am walking at a speed of 7 miles every 2 hours. Determine a table which describes the distance from your house, D, given the number of hours, t, after the realization of your speed. Use 0.25, 0.5, 0.75... up to 2.5 hours for your table. Then determine the symbolic linear function which cooresponds.

You're awesome! Now we're going to review how to graph a linear function given its symbolic representation.

Suppose $f(x) = \frac{2}{3}x + 4$. By far, the easiest way to graph this is to remember that the y-int is $(0, 4)$ and the *slope* is $\frac{2}{3}$. Thus we may start by marking the point (0,4) on our graph and then going up 2 and over 3 to find another point on the line, in this case (3,6). We then simply draw the line through these two points and voila, we're done! Go ahead and draw this graph here:

Now, next to the graph make a table of values which also represents a *portion* of this function. Why do I use the word portion?

Let's practice this by graphing the following and then making a table of values.

$f(x) = -\frac{1}{2}x - 2$

$g(x) = 3x + \frac{3}{4}$

$h(x) = -1 + \frac{3}{4}x$

Let's go back and real quick give verbal descriptions of each of these functions.

Here at PCC, resident tuition cost is currently at \$88 per credit (not including the per credit fees) and there is a fixed fee of \$19 each term for college service and transportation. Not including the per credit fees (which I haven't described), determine the symbolic representation of the function which gives the cost, C, of going to school here based upon the number of credits, n, which you are signed up for. Then graph this function on an appropriate coordinate plane and give a table of values which describes 5 ordered pairs which this function takes on.

I mentioned **slope** earlier. What is slope?

So, if we know two points on a line we may calculate the slope of the line using the equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Real quick, what is the slope of the line passing through the points (1,3) and (5,-2)?

What about the line passing through (-3,-2) and (6,-2)?

Through (-7,4) and (3,1)?

What about the line perpendicular to a line which passes through the points (1,1) and (2,-1)?

What about a line passing through $\left(-\frac{3}{4}, \frac{1}{4}\right)$ and $\left(-\frac{1}{4}, \frac{5}{4}\right)$?

Of the above lines, which lines are increasing and which are decreasing? Write what it means for a line to be increasing in your own words.

The line passing through $(a, 3b)$ and $(3a, 5b)$?

Copy down the figure I put on the board which gives a way of guestimating the slope of a line.

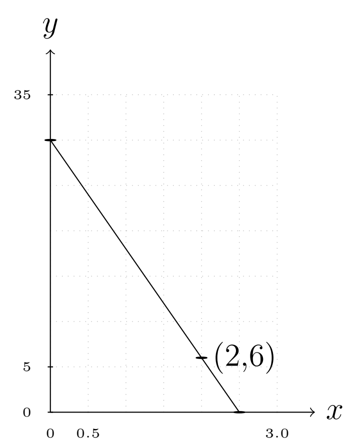
I'm going to draw some graphs on the board. Copy them down and then figure out the slope of the line. Then see if you can figure out the function which describes the lines. [hint: $f(x) = mx + b$, what does the b work out to be?]

What if I told you that a line has a $y - int = (0, 3)$ and a slope of $m = -\frac{3}{7}$. What is the function describing this line?

What about the function describing the line with $y - int = \left(0, -\frac{3}{5}\right)$ and slope of $m = 1$?

How about the function with $y - int = 218.3$ and slope of $m = 0$?

The distance y in miles that an athlete riding a bicycle is from home after x hours is shown in the following graph.



a) Find the y-intercept and interpret what it means in context of the story.

b) The graph passes through the point (2,6). Discuss the meaning of this point.

c) Find the slope-intercept form of this line and write it using function notation. Iterpret the slope as a rate of change in context of the story.

If a sample of gas is heated, it will expand. The function $V(t) = 0.183t + 50$ gives the volume V of a sample of helium in cubic inches at a temperature of t degrees Celsius.

a) What is the slope of the graph of V?

b) Interpret the slope as a rate of change in context of the story.

c) Graph the function on an appropriate coordinate plane.

Alright, we've covered some contextual things, dealt with graphing a line, what slope is and determining a linear function when given an initial value (y-intercept) and a rate of change (slope). But, what if we want to determine a linear function without knowing the y-intercept? Like, knowing two points on the line or one point and the slope? Well, to accomplish this we need the $point - slope$ form of the equation of a line. I'm going to start this discussion a little different than you have probably ever seen this before.

Remember our idea for what a linear function is? It is a function with a constant rate of change so that as x increases by a set amount, y will always increase (or decrease) by the same amount. That is, given any two points on the graph, the slope between those points will remain the same. So if I have points (x_1, y_1), (x_2, y_2) and (a, b) on the same graph then the following must be true:

$$\frac{b-y_2}{a-x_2} = \frac{b-y_1}{a-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

That is, the slope between each set of points must be the same. More generally, I could say that if I were given a *variable point* (x, y) on the graph of a linear function and I know that (x_1, y_1) and (x_2, y_2) are points which satisfy the function, then

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Then, all I need to do is multiply both sides by $x - x_1$ in order to obtain the *point slope equation of a line*:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1) \text{ or } y - y_1 = m(x - x_1)$$

where the second equation is simply obtained by substituting m in for the slope.
Now, for our purposes we need to be thinking about functions and need to come up with a point-slope form of a linear function. To do this we simply add y_1 to both sides to get

$$y = m(x - x_1) + y_1 \text{ and by replacing } y \text{ with } f(x)\text{: } f(x) = m(x - x_1) + y_1.$$

Let's get some practice using this form:

Write a function which describes a line passes through the point (3,2) and has a slope of $m = 2$.

What is the function which passes through the points (-1,4) and (3,-2).

How about the function which passes through (-2,3) and (6,-1)?

And the function which passes through (1,4) and is perpendicular to $g(x) = -\frac{2}{3}x - 1$?

This last problem brings up a couple of quick topics to review. When are two lines perpendicular? How will their slopes be related?

When are two lines parallel? How will their slopes be related?

What is the slope of a horizontal line? A vertical line?

What is the equation of a horizontal line passing through (2,5)?

What is the equation of a vertical line passing through (2,5)?

In your own words, describe why these are the correct equations.

Hat sizes can be modeled by a linear function. If the circumference of a person's head is $21\frac{7}{8}$ inches, the hat size is 7 and if the circumference is 25 inches, the hat size is 8.

a) Find $H(c) = mc + b$ so that H calculates the hat size for a head with circumference c.

b) Sketch the linear function $H(c)$.

c) Identify the c-intercept and the H-intercept and interpret their meaning in context of the story.

d) Suppose a person's head has a circumference of 23 inches. What is the hat size to the nearest eighth?