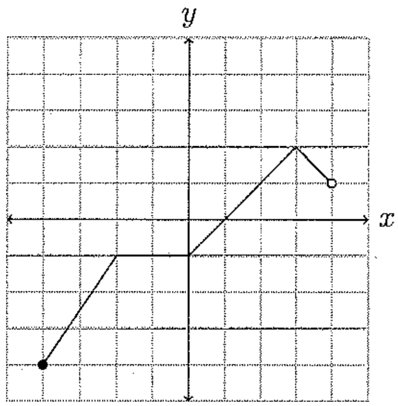
Math 111 WS 2, Graphical Behavior of Functions

Name: Solutions

1. For each of the following graphs, determine where the function is increasing, decreasing, and constant. State any local and absolute maximums or minimums along with the location of the local/absolute maximums and minimums. Find any inflection points and state where the function is concave up and concave down.

a.

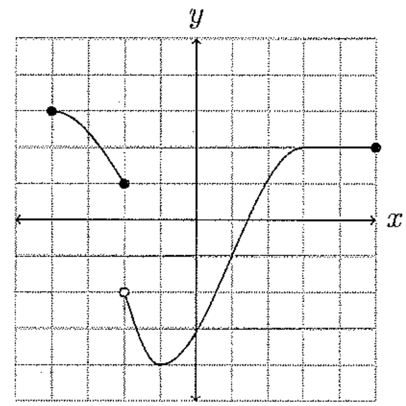


$y = f(x)$

local min of -1 on $(-2,0]$
local max of -1 on $[-2,0)$
local max of 2 at $x=3$
abs min of -4 at $x=-4$
abs max of 2 at $x=3$
inc: $(-4,-2)\cup(0,3)$
dec: $(3,4)$ const: $(-2,0)$

no concave up or down
no inflections

c.

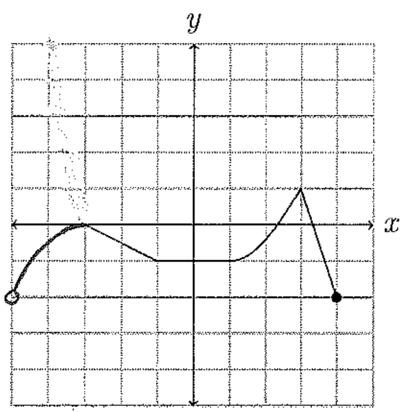


$y = h(x)$

local min of -4 at $x=-1$
local min of 2 on $(3,5)$
local max of 2 on $[3,5)$
abs min of -4 at $x=-1$
abs max of 3 at $x=-4$
inc: $(-1,3)$ dec: $(-4,-1)$
const: $(3,5)$

inf pnt: $(1,-1)$
con up: $(-2,1)$
con down: $(-4,-2)\cup(1,3)$

b.

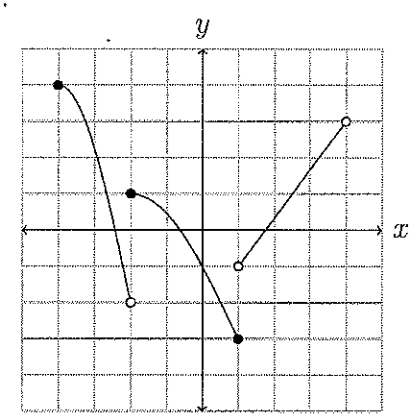


$y = g(x)$

local min of -1 on $[-1,1]$
local max of 0 at $x=-3$
loc. max of 1 at $x=3$
loc max of -1 on $(-1,1)$
abs min of -2 at $x=4$
abs max of 1 at $x=3$
inc: $(-5,-3)\cup(1,3)$
dec: $(-3,-1)\cup(3,4)$

const: $(-1,1)$
no inf. pnts.
con up: $(1,3)$
con dwn: $(-5,-3)$

d.



$y = j(x)$

no local mins or maxs.
abs min of -3 at $x=1$
abs max of 4 at $x=-4$
inc: $(1,4)$
dec: $(-4,-2)\cup(-2,1)$
const: nowhere

no inf pnt.
con up: nowhere
con dwn: $(-4,-2)\cup(-2,1)$

2. Graph the following functions and determine the local and absolute minimums and maximums along with their locations and then state where the function is increasing, decreasing, or constant. Estimate the inflection points and then state where the function is concave up or concave down.

a. $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 3$

local min of -24 at $x=3$
local max of $61.\bar{3}$ at $x=-5$
no abs min or max
inc: $(-\infty,-5)\cup(3,\infty)$
dec: $(-5,3)$ const: nowhere
Inf pnt: $\approx(-1,18.\bar{6})$
con dwn: $(-\infty,-1)$
con up: $(-1,\infty)$

b. $g(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x - 4$

local mins: $-16.\bar{6}$ at $x=-2$, -6.25 at $x=3$
local max: -0.917 at $x=1$
abs min: $-16.\bar{6}$ at $x=-2$
no abs max
inc: $(-2,1)\cup(3,\infty)$
dec: $(-\infty,-2)\cup(1,3)$
inf pnts: $\approx(-1,-11.583)$ & $(2,-3.\bar{3})$
con up: $(-\infty,-1)\cup(2,\infty)$
con dwn: $(-1,2)$

c. $h(x) = 6x^2 - 3x - 3$

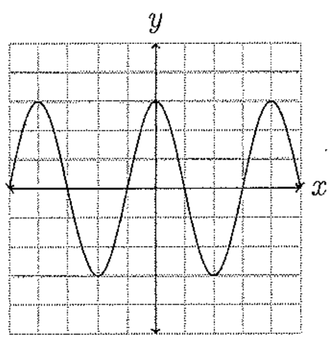
local/abs min of -3.375 at $x=.25$
no local or abs max
inc: $(.25,\infty)$ dec: $(-\infty,.25)$
inf pnts: none
con up: $(-\infty,\infty)$ con down: nowhere

d. $j(x) = 2x^5 - 3x^2 - 3$

local min of -4.28 at $x\approx 0.843$
local max of -3 at $x=0$
no abs max or min
inc: $(-\infty,0)\cup(0.843,\infty)$ dec: $(0,0.843)$
inf pnt $\approx(.5,-3.704)$
con up: $(.5,\infty)$ con dwn: $(-\infty,.5)$

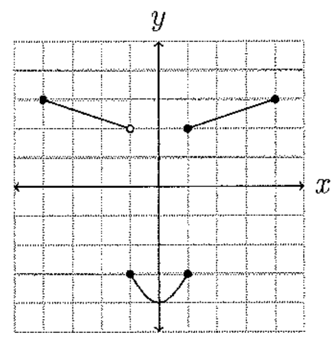
3. Determine whether the following functions are even, odd, or neither by looking at their graphs.

a.



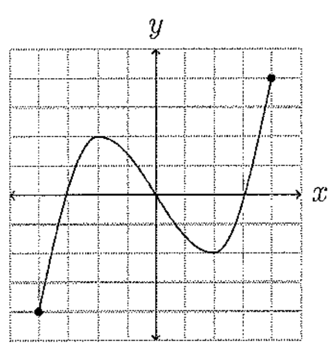
even

c.



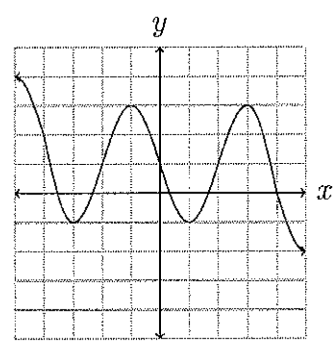
neither

b.



odd

d.



neither

4. Determine whether the following functions are even, odd, or neither by checking the definitions.

a. $f(x) = 3x^2 - 4$

$f(-x) = 3(-x)^2 - 4$
$= 3x^2 - 4$
$= f(x)$

Thus f is even.

c. $h(x) = 2x^4 + 7x^8$

$h(-x) = 2(-x)^4 + 7(-x)^8$
$= 2x^4 + 7x^8$
$= h(x)$

Thus h is even

b. $g(x) = -5x^5 + 2x$

$g(-x) = -5(-x)^5 + 2(-x)$
$= 5x^5 - 2x \neq g(x)$
$= -(-5x^5 + 2x)$
$= -g(x)$

Thus g is odd.

d. $j(x) = 7x^3 - 10x^2 + x$

$j(-x) = 7(-x)^3 - 10(-x)^2 + (-x)$
$= -7x^3 - 10x^2 - x \neq j(x)$
$= -(7x^3 + 10x^2 + x) \neq -j(x)$

Thus j is neither even nor odd.

e. $k(x) = \dfrac{-x^3+x}{2x^2-5x^4}$

$$k(-x) = \frac{-(-x)^3+(-x)}{2(-x)^2-5(-x)^4}$$

$$= \frac{x^3-x}{2x^2-5x^4} \neq k(x)$$

$$= -\frac{-x^3+x}{2x^2-5x^4} = -k(x)$$

Thus k is odd

g. $m(x) = \dfrac{x^4+x^2}{3x^2+5}$

$$m(-x) = \frac{(-x)^4+(-x)^2}{3(-x)^2+5}$$

$$= \frac{x^4+x^2}{3x^2+5}$$

$$= m(x)$$

Thus m is even

f. $l(x) = \dfrac{2x^7+3x^3+x}{x^5-x^3}$

$$l(-x) = \frac{2(-x)^7+3(-x)^3-x}{(-x)^5-(-x)^3}$$

$$= \frac{-2x^7-3x^3-x}{-x^5+x^3} \neq l(x)$$

$$= \frac{-(2x^7+3x^3+x)}{-(x^5-x^3)}$$

$$= \frac{2x^7+3x^3+x}{x^5-x^3}$$

$$= l(x)$$

Thus l is even

h. $n(x) = \dfrac{x^4-x^3}{x^6-2x+1}$

$$n(-x) = \frac{(-x)^4-(-x)^3}{(-x)^6-2(-x)+1}$$

$$= \frac{x^4+x^3}{x^6+2x+1} \neq n(x)$$

$$= \frac{-(-x^4-x^3)}{-(-x^6-2x-1)}$$

$$= \frac{-x^4-x^3}{-x^6-2x-1} \neq -n(x)$$

$$\neq n(x)$$

Thus n is neither even nor odd.