Math 95 WS 4 Factoring and Solving Quadratic Equations Review Name: ______________________________

§5.6, 5.7 We will be reviewing how to solve quadratic equations. In order to do this it is very, very helpful to know how to factor. Factoring is also key in a number of other algebraid operations. For these reasons let's begin by reviewing the following factoring strategies:
Factoring out the GCF, Factoring by Grouping, Factoring a trinomial when the leading coefficient is 1, Factoring a trinomial when the leading coefficient is *not* 1 and Factoring Special Forms.

Factoring by the Greatest Common Factor (GCF):
Start by identifying the largest factor that divides each term of the polynomial. The number piece of the factor should divide all other coefficients and the variable piece should contain the lowest power of a *shared* variable.
NOTE: We will only include a variable in the GCF if that variable shows up in every term!

Example: Factor out the GCF of $36x^3y + 60x^2$.
Answer: We notice that 12 is the largest number that divides both 36 and 60. This is our number part. Looking at our variables, we see that each term has an x. This is our only shared variable. The lowest power for x is x^2. This is our variable part. Factoring this out, we get:

$$36x^3y + 60x^2 = 12x^2(3xy + 5)$$

Notice that if we distributed $12x^2$ into $(3xy + 5)$ we would get back our original expression!

Factoring by Grouping-Four Terms: If we have four terms and we cannot simplify any further, it is often a good idea to group together two sets of two terms that have something in common. Ideally, your sets can be factored using the GCF method.

Example: Factor completely $2x^3 + 3x^2 - 2xy - 3y$.
Answer: We notice that both the first two terms have x^2 in common. The second two terms have a y in common. We group these and factor accordingly:

$$\begin{aligned} 2x^3 + 3x^2 - 2xy - 3y &= [2x^3 + 3x^2] + [-2xy - 3y] \\ &= x^2[2x + 3] - y[2x + 3] \\ &= [x^2 - y][2x + 3] \end{aligned}$$

NOTE: We proceeded from line 2 by factoring out the GCF $(2x + 3)$!

Factoring a trinomial with leading coefficient of 1:
This method basically explains how to *undo* FOILing! For instance, we know that

$$(x+3)(x-4) = x^2 - x - 12.$$

So, that means that

$$x^2 - x - 12 = (x+3)(x-4)!$$

What we are seeing here is that, although $x^2 - x - 12$ has no gcf, it can still be factored! So, how are we going to go about factoring trinomials of this sort when they have no gcf? The technique is that we look at the constant and try and find two factors of it which add up to the coefficient of the middle term.

Example: Factor $x^2 + 2x - 15$
Answer: We write a factor tree for -15:

$$\begin{array}{cc} & -15 \\ / & \backslash \\ 1 & 15 \\ 3 & 5 \end{array}$$

Since -3 and 5 multiply to be -15 and add to be positive 2, these are the numbers we are going to use.
NOTE: Watch your signs carefully! 3 and -5 won't work because $3 - 5 = -2$ not positive 2!
So $x^2 + 2x - 15 = (x-3)(x+5)$. We just make the two sets of parenthesis, put an x into each one and then put in the numbers we just found. That's it!

Factoring a trinomial with leading coefficient not equal to 1:
This method is often referred to as the ac-method. This is because the standard form of a trinomial is $ax^2 + bx + c$ and the first step in factoring a trinomial of this form is to multiply the a and the c together. We will then find two factors of the product ac that add up to the middle term, like in the last example, but this time we cannot just put the numbers in the parenthesis. This time we will break the middle term up into two terms using the numbers we find and then we will factor by grouping.

Example: Factor $6x^2 + 7x - 3$
Answer: We begin by multiplying 6 and -3 and then looking for two factors of their product, -18, which add up to 7.

$$\begin{array}{cc} & -18 \\ / & \backslash \\ 1 & 18 \\ 2 & 9 \\ 3 & 6 \end{array}$$

Since -2 and 9 add up to positive 7, these are the numbers we are going to use. This time, we will re-write $7x$ as $7x = -2x + 9x$ and then factor by grouping.

$$\begin{aligned} &6x^2 + 7x - 3 \\ =&6x^2 - 2x + 9x - 3 \\ =&2x(3x-1) + 3(3x-1) \\ =&(3x-1)(2x+3) \end{aligned}$$

Special Forms:
There are a few common forms to watch out for when factoring. They are as follows:

$$\begin{aligned} x^2 - y^2 &= (x-y)(x+y) \\ x^3 - y^3 &= (x-y)(x^2 + xy + y^2) \\ x^3 + y^3 &= (x+1)(x^2 - xy + y^2) \\ x^2 + 2xy + y^2 &= (x+y)^2 \end{aligned}$$

Example: Factor $x^2 - 25$
Answer: We notice that $x^2 - 25 = x^2 - 5^2$ looks like the form $x^2 - y^2$ where $y = 5$. Using the above formula, we factor the difference of squares to get:

$$x^2 - 25 = (x-5)(x+5)$$

How to Verify: Checking your work is easy. Just multiplying out your answers should get you back to the original problem! Good luck and happy factoring!

IMPORTANT NOTE: When given any polynomial to factor, ALWAYS pull out the GCF FIRST!

Practice: Try factoring the following polynomials!

1. $12y - 24xy^2$
2. $10x^2y^3 + 5xy^2$
3. $8xy - 2x + 12y - 3$
4. $-x^3 + 6yx + x^2 - 6y$
5. $16xy - 2x^3 + 32y - 4x^2$
6. $x^2 + 5x + 6$

7. $x^2 + 6x + 9$

8. $5x^2 - 2x - 16$

9. $6x^2 - 17x + 12$

10. $x^2 - 16$

11. $x^3 - 8$

12. $27x^3 + 1$

13. $x^4 - 1$ Hint: $x^4 - 1 = (x^2)^2 - 1^2$

    The answers to these problems are on the next page!

ANSWERS:

1. $12y - 24xy^2 = 12y(1 - xy)$
2. $10x^2y^3 + 5xy^2 = 5xy^2(2xy + 1)$
3. $8xy - 2x + 12y - 3 = (2x + 3)(4y - 1)$
4. $-x^3 + 6yx + x^2 - 6y = (-x + 1)(x^2 - 6y)$
5. $16xy - 2x^3 + 32y - 4x^2 = 2(x + 2)(8y - x^2)$
6. $x^2 + 5x + 6 = (x + 2)(x + 3)$
7. $x^2 + 6x + 9 = (x + 3)^2$
8. $5x^2 - 2x - 16 = (x - 2)(5x + 8)$
9. $6x^2 - 17x + 12 = (3x - 4)(2x - 3)$
10. $x^2 - 16 = (x - 4)(x + 4)$
11. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
12. $27x^3 + 1 = (3x + 1)(9x^2 - 3x + 1)$
13. $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x + 1)(x - 1)(x^2 + 1)$

Now let's review how to solve polynomial equations. We may introduce a couple of new concepts. The first thing we need is a definition:

Quadratic Equation: An equation that can be written in *standard form*

$$ax^2 + bx + c = 0$$

where a, b and c are real numbers and $a = ne0$. A quadratic equation is also known as a **second-degree polynomial equation.**

The next group of problems requires some factoring method that we've reviewed in order to solve the given quadratic equation. Go ahead and give them a shot.

14. $3x^2 - 2x = 0$

15. $2x^2 + 7x - 4 = 0$

16. $x^2 = 6x - 9$

17. $x(x-3)=0$

18. $(x-3)(x+8)=0$

19. $x^2+7x=18$

20. $3x^2=x+4$

21. $x(3x+8)=-5$

22. $(x+3)(3x+5) = 7$

Let's remember the square root property is simply the fact that if $u^2 = d$ then $u = \pm\sqrt{d}$ so long as $d \geq 0$.

Why is it that we put the $\pm$ before the square root?

Use this fact to solve the following:

23. $4x^2 = 20$

24. $(x-1)^2 = 5$

25. $x^2 + 4x + 4 = 25$

26. $x^2 + 2x + 1 = 5$

Now, what if we have a quadratic equation that cannot be factored and we cannot use the square root property?! Well, we have a special formula to solve such equations! We will learn in the next week or two where this formula comes from, but for now I'm just going to give it to you and then you get to use it! Here's how it goes:

Say you have a quadratic equation in standard form

$$ax^2 + bx + c = 0$$

and you need to solve for x. Well, then we're going to have the following solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let me clarify now: YOU NEED TO MEMORIZE THIS FORMULA! Write it down three times right before going to sleep. If you don't remember it the next day, write it down before going to sleep again and so forth until you're having wonderful math dreams about it!

Let's see how it works. Do the following problems:

27. $2x^2 = 6x - 1$

28. $3x^2 - 5x + 1 = 0$

29. $3x^2 = 6x + 5$

30. $4x^2 + 2x + 3 = 0$

Whoa! What happened on that last one? What does this tell us?

Okay, let's review again. Write down each of the strategies we've used so far and then set up a system to know which technique to use for which situations!

So far these are all concepts that we covered last term. Let's introduce now how to solve a few polynomials of higher degree.

Suppose I want to solve $x^3 - 5x^2 - x + 5 = 0$. Is there some way to use factoring to still be able to solve this equation even though it is NOT a quadratic? What is it? Go ahead and use this technique to find the solution.

What is the technique necessary for solving $16x^4 - 64x^3 + 64x^2 = 0$? Go ahead and find the solution here as well.

How might I approach solving $x^4 - 5x^2 + 4 = 0$? Find the solution of course.

Here are a couple extra for you to try:

31. $6x^5 + 2x^3 - 9x^3 - 3x$

32. $5x^4 - 27x^2 + 10$

33. $x^3 + 4x^2 - 21x$