Math 252 In Class Worksheet 5

Definition of a Definite Integral: If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$. We let $x_0 = a, x_1, ..., x_n = b$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, ..., x_n^*$ be any **sample points** in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists. If it does exist we say that f is **integrable** on $[a, b]$.

The sum $\sum_{i=1}^{n} f(x_i^*)\Delta x$ is called a **Riemann sum** after German mathematician Bernhard Riemann (1826-1866).

Helpful things:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

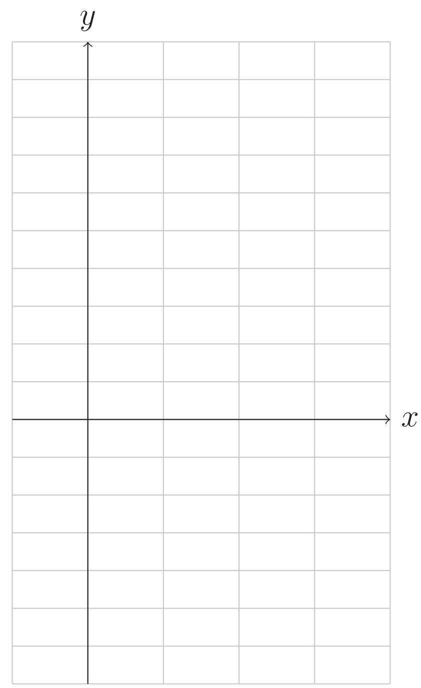
$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} c = nc$$

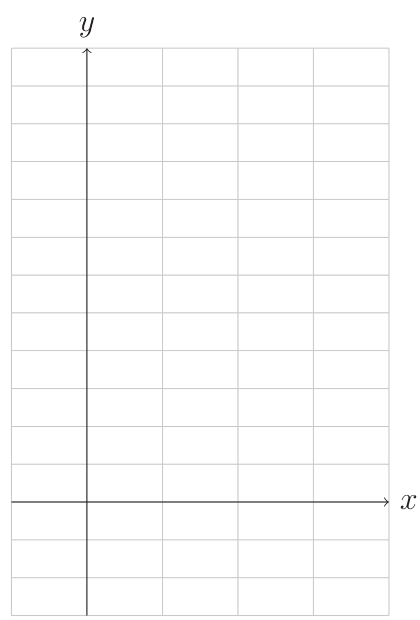
$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

1. Evaluate $\int_0^3 x^3 - 6dx$.



2. Evaluate $\int_1^3 \frac{1}{2}x^2 + 3xdx$.



3. Evaluate $\int_0^2 2x^3 - 4x dx$.

4. Evaluate $\int_{-1}^2 3x^2 + x - 5$.

5. Evaluate the following integrals by interpreting them as an area under the curve.

a. $\int_0^1 \sqrt{1-x^2}dx$

c. $\int_0^{10} |x-5|dx$

b. $\int_0^3 (x-1)dx$

d. $\int_{-1}^3 (3-2x)dx$

More helpful stuff:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b c\,dx = c(b-a), \text{ where } c \text{ is any constant.}$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

1. Use the properties of integrals to evaluate $\int_0^1 (4+3x^2)dx$ Noting that $\int_0^1 x^2dx = \dfrac{1}{3}$.

2. If it is known that $\int_0^{10} f(x)dx = 17$ and $\int_0^8 f(x)dx = 12$, find $\int_8^{10} f(x)dx$.